Homework Assignment #5

Due Time/Date

This assignment is due 1:20PM Tuesday, April 2, 2024. Late submission will be penalized by 20% for each working day overdue.

How to Submit

Please use a word processor or scan hand-written answers to produce a single PDF file and name the file according to this pattern: "b107050xx-hw5". Upload the PDF file to the NTU COOL site for this course. You may discuss the problems with others, but copying answers is strictly forbidden.

Problems

(Note: problems marked with "Exercise X.XX" or "Problem X.XX" are taken from [Sipser 2006, 2013] with probable adaptation.)

1. (Exercise 2.1; 10 points) Consider the following CFG discussed in class, where for convenience the variables have been renamed with single letters.

$$
\begin{aligned}
E &\rightarrow E + T \mid T \\
T &\rightarrow T \times F \mid F \\
F &\rightarrow (E) \mid a
\end{aligned}
$$

   Give (leftmost) derivations and the corresponding parse trees for the following strings.

   (a) $a + (a \times a)$

   (b) $((a) \times a)$

2. (Exercise 2.4; 10 points) Give CFGs that generate the following languages. In all parts the alphabet Σ is $\{0, 1\}$.

   (a) $\{w \mid \text{the length of } w \text{ is a multiple of } 3\}$

   (b) $\{w \mid w = w^R, \text{ that is, } w \text{ is a palindrome}\}$

3. (Exercise 2.6d; 10 points) Give a CFG that generates the language $\{x_1 \# x_2 \# \cdots \# x_k \mid k \geq 1,$ each $x_i \in \{a, b\}^*$, and for some i and j, $x_i = x_j^R\}$.

4. (Problem 2.33; 20 points) Let $\Sigma = \{a, b\}$. Give a CFG generating the language of strings with twice as many a's as b's (no restriction is imposed on the order in which the input symbols may appear). Prove that the CFG is correct.

5. (Exercise 2.8 adapted; 10 points) Show that the string "`the boy sees a girl with a telescope`" has two different leftmost derivations in the following CFG.

$$
\begin{aligned}
\langle\text{SENTENCE}\rangle &\rightarrow \langle\text{NOUN-PHRASE}\rangle\langle\text{VERB-PHRASE}\rangle \\
\langle\text{NOUN-PHRASE}\rangle &\rightarrow \langle\text{CMPLX-NOUN}\rangle \mid \\
&\qquad \langle\text{CMPLX-NOUN}\rangle\langle\text{PREP-PHRASE}\rangle \\
\langle\text{VERB-PHRASE}\rangle &\rightarrow \langle\text{CMPLX-VERB}\rangle \mid \\
&\qquad \langle\text{CMPLX-VERB}\rangle\langle\text{PREP-PHRASE}\rangle \\
\langle\text{PREP-PHRASE}\rangle &\rightarrow \langle\text{PREP}\rangle\langle\text{CMPLX-NOUN}\rangle \\
\langle\text{CMPLX-NOUN}\rangle &\rightarrow \langle\text{ARTICLE}\rangle\langle\text{NOUN}\rangle \\
\langle\text{CMPLX-VERB}\rangle &\rightarrow \langle\text{VERB}\rangle \mid \langle\text{VERB}\rangle\langle\text{NOUN-PHRASE}\rangle \\
\langle\text{ARTICLE}\rangle &\rightarrow \texttt{a} \mid \texttt{the} \\
\langle\text{NOUN}\rangle &\rightarrow \texttt{boy} \mid \texttt{girl} \mid \texttt{flower} \mid \texttt{telescope} \\
\langle\text{VERB}\rangle &\rightarrow \texttt{touches} \mid \texttt{likes} \mid \texttt{sees} \\
\langle\text{PREP}\rangle &\rightarrow \texttt{with}
\end{aligned}
$$

6. (Exercise 2.9; 20 points) Give a CFG that generates the language

$$A = \{a^i b^j c^k \mid i = j \text{ or } j = k \text{ where } i, j, k \geq 0\}.$$

Is your grammar ambiguous? Why or why not?

7. (Exercise 2.14; 20 points) Convert the following CFG (where A is the start variable) into an equivalent CFG in Chomsky normal form, using the procedure given in Theorem 2.9.

$$
\begin{aligned}
A &\rightarrow BAB \mid B \mid \varepsilon \\
B &\rightarrow 0B1 \mid \varepsilon
\end{aligned}
$$