Homework Assignment #6

Due Time/Date

This assignment is due 2:20PM Tuesday, April 16, 2024; however, to be better prepared for the midterm exam on April 9, you should try to complete it before the exam. Late submission will be penalized by 20% for each working day overdue.

How to Submit

Please use a word processor or scan hand-written answers to produce a single PDF file and name the file according to this pattern: "b107050xx-hw6". Upload the PDF file to the NTU COOL site for this course. You may discuss the problems with others, but copying answers is strictly forbidden.

Problems

(Note: problems marked with "Exercise X.XX" or "Problem X.XX" are taken from [Sipser 2006, 2013] with probable adaptation.)

1. (Exercise 2.2; 10 points)
   (a) Use the languages $A = \{a^n b^n c^m \mid m, n \geq 0\}$ and $B = \{a^m b^n c^n \mid m, n \geq 0\}$, together with the fact that $\{a^n b^n c^n \mid m, n \geq 0\}$ is not context free, to show that the class of context-free languages is not closed under intersection.
   (b) Use the preceding part and DeMorgan's law to show that the class of context-free languages is not closed under complementation.

2. (Exercise 2.5; 20 points) Give informal descriptions and state diagrams of pushdown automata for the following languages. In all parts the alphabet Σ is $\{0, 1\}$.
   (a) $\{w \mid$ the length of w is a multiple of $3\}$
   (b) $\{w \mid w$ is a palindrome, that is, $w = w^R\}$

3. (Exercise 2.12; 10 points) Convert the following CFG to an equivalent PDA, using the procedure given in Theorem 2.20.

$$\begin{aligned} E &\to E + T \mid T \\ T &\to T \times F \mid F \\ F &\to (\ E\ ) \mid a \end{aligned}$$

4. (Problem 2.39; 20 points) Let $G = (V, \Sigma, R, \langle \text{STMT} \rangle)$ be the following grammar.

$$\begin{aligned} \langle \text{STMT} \rangle &\to \langle \text{ASSIGN} \rangle \mid \langle \text{IF-THEN} \rangle \mid \langle \text{IF-THEN-ELSE} \rangle \\ \langle \text{IF-THEN} \rangle &\to \texttt{if condition then } \langle \text{STMT} \rangle \\ \langle \text{IF-THEN-ELSE} \rangle &\to \texttt{if condition then } \langle \text{STMT} \rangle \texttt{ else } \langle \text{STMT} \rangle \\ \langle \text{ASSIG} \rangle &\to \texttt{a := 1} \end{aligned}$$

$$\Sigma = \{\texttt{if}, \texttt{condition}, \texttt{then}, \texttt{else}, \texttt{a := 1}\}$$

$$V = \{\langle \text{STMT} \rangle, \langle \text{IF-THEN} \rangle, \langle \text{IF-THEN-ELSE} \rangle, \langle \text{ASSIG} \rangle\}$$

G is a *natural-looking* grammar for a fragment of a programming language, but G is ambiguous.

(a) Show that G is ambiguous.

(b) Give a new unambiguous grammar for the same language.

5. (Problem 2.32; 20 points) Let $A/B = \{w \mid wx \in A \text{ for some } x \in B\}$. Show that, if A is context free and B is regular, then A/B is context free.

6. (10 points) Prove (using the pumping lemma) that $\{a^m b^n c^{m \times n} \mid m, n \geq 1\}$ is not context free.

7. (Problem 2.57; 10 points) Let $A = \{wtw^R \mid w, t \in \{0,1\}^* \text{ and } |w| = |t|\}$. Prove that A is not context free.