Homework Assignment #8

Due Time/Date

This assignment is due 2:20PM Tuesday, May 7, 2024. Late submission will be penalized by 20% for each working day overdue.

How to Submit

Please use a word processor or scan hand-written answers to produce a single PDF file and name the file according to this pattern: "b107050xx-hw7". Upload the PDF file to the NTU COOL site for this course. You may discuss the problems with others, but copying answers is strictly forbidden.

Problems

(Note: problems marked with "Exercise X.XX" or "Problem X.XX" are taken from [Sipser 2013] with probable adaptation.)

1. (10 points) Give a formal definition (with a state diagram) of a Turing machine that, given a string of an even length as the input, splits the input string into two halves and add a # in the middle to separate the two substrings. The input alphabet is $\{0, 1\}$.

2. (Problem 3.10; 20 points) Let $c_1x^n + c_2x^{n-1} + \cdots + c_nx + c_{n+1}$ be a polynomial with a root at $x = x_0$. Let $c_{\max}$ be the largest absolute value of a c_i. Show that

$$|x_0| < (n+1)\frac{c_{\max}}{|c_1|}.$$

3. (Problem 3.11; 20 points) Show that single-tape TMs that cannot write on the portion of the tape containing the input string recognize only regular languages.

4. (Problem 3.12; 20 points) Show that every infinite Turing-recognizable language has an infinite decidable subset.

5. (20 points) Let $A = \{\langle M, N\rangle \mid M \text{ is a PDA and } N \text{ is a DFA such that } L(M) \subseteq L(N)\}$. Show that A is decidable.

6. (Exercise 4.4; 10 points) Let $A\varepsilon_{\text{CFG}} = \{\langle G\rangle \mid G \text{ is a CFG that generates } \varepsilon\}$. Show that $A\varepsilon_{\text{CFG}}$ is decidable.