Homework Assignment #9

Due Time/Date

This assignment is due 2:20PM Tuesday, May 14, 2024. Late submission will be penalized by 20% for each working day overdue.

How to Submit

Please use a word processor or scan hand-written answers to produce a single PDF file and name the file according to this pattern: "b107050xx-hw7". Upload the PDF file to the NTU COOL site for this course. You may discuss the problems with others, but copying answers is strictly forbidden.

Problems

(Note: problems marked with "Exercise X.XX" or "Problem X.XX" are taken from [Sipser 2013] with probable adaptation.)

1. (Exercise 4.7; 10 points) Let B be the set of all infinite sequences over $\{0, 1\}$. Show that B is uncountable, using a proof by diagonalization.

2. (Problem 4.12; 10 points) Let A be a Turing-recognizable language consisting of descriptions of Turing machines, $\{\langle M_1\rangle, \langle M_2\rangle, \ldots\}$, where every M_i is a decider. Prove that some decidable language D is not decided by any decider M_i whose description appears in A. (Hint: you may find it helpful to consider an enumerator for A.)

3. (Problem 4.14; 20 points) Let $C = \{\langle G, x\rangle \mid G \text{ is a CFG and } x \text{ is a substring of some } y \in L(G)\}$. Show that C is decidable. (Hint: an elegant solution to this problem uses the decider for E_{CFG}.)

4. (Problem 4.16; 10 points) Let $PAL_{\text{DFA}} = \{\langle M\rangle \mid M \text{ is a DFA that accepts some palindrome}\}$. Show that PAL_{DFA} is decidable. (Hint: Theorems about CFLs are helpful here.)

5. (Problem 4.31; 20 points) Let $INFINITE_{\text{PDA}} = \{\langle M\rangle \mid M \text{ is a PDA and } L(M) \text{ is infinite}\}$. Show that $INFINITE_{\text{PDA}}$ is decidable.

6. (Exercise 5.1; 10 points) Show that EQ_{CFG} is undecidable.

7. (Exercise 5.4; 20 points) If A is reducible to B and B is a regular language, does that imply that A is a regular language? Why or why not?