Homework Assignment #10: Programming Exercise #2

Note

This assignment constitutes 4% of your grade and is due 2:10PM Friday, June 13, 2011. Please write/type your answers/code on A4 (or similar size) paper. Drop your homework by the due time in Yih-Kuen Tsay's mail box on the first floor of Management College Building II. Late submission will be penalized by 20% for each working day overdue. You may discuss the problems with others, but copying answers/code is strictly forbidden.

Your work will be graded according to its correctness and presentation. Specifically, you should demonstrate evidences showing that your program is correct. You should also organize and document your program in such a way that your classmates, for example, can understand it.

Problem

Solve Problem A "To Add or to Multiply" or Problem J "Pyramids" of the 2011 Annual ACM International Collegiate Programming Contest World Finals (see the appended).

Please prepare an input file with more interesting cases and test your program on the input. In the documentation of your program, you should describe how you have applied the algorithmic techniques, in particular design by induction, learned in class.

Problem A
To Add or to Multiply
Problem ID: addmul

The Industrial Computer Processor Company offers very fast, special purpose processing units tailored to customer needs. Processors of the a-C-m family (such as the 1-C-2 and the 5-C-3) have an instruction set with only two different operations:

`A` add a
`M` multiply by m

The processor receives an integer, executes a sequence of `A` and `M` operations (the program) that modifies the input, and outputs the result. For example, the 1-C-2 processor executing the program `AAAM` with the input 2 yields the output 10 (the computation is $2 \to 3 \to 4 \to 5 \to 10$), while the 5-C-3 processor yields 51 with the same program and input ($2 \to 7 \to 12 \to 17 \to 51$).

You are an a-C-m programmer assigned to a top secret project. This means that you have not been told the precise computation your program should perform. But you are given particular values p, q, r, and s and the following conditions:

1. The input is guaranteed to be a number between p and q.
2. The output must be some number between r and s.

Given an a-C-m processor and the numbers p, q, r, and s, your job is to construct the shortest a-C-m program which, for every input x such that $p \leq x \leq q$, yields some output y such that $r \leq y \leq s$. If there is more than one program of minimum length, choose the one that come first lexicographically, treating each program as a string of `A`s and `M`s.

Input

The input contains several test cases. Each test case is given by a line with the six integers a, m, p, q, r, and s as described above ($1 \leq a, m, p, q, r, s \leq 10^9$, $p \leq q$ and $r \leq s$).

The last test case is followed by a line with six zeros.

Output

For each test case, display its case number followed by the best program as described above. Display the word "`empty`" if the best program uses no operations. Display the word "`impossible`" if there is no program meeting the specifications.

Display the program as a sequence of space-separated strings, alternating between strings of the form "n`A`" and strings of the form "n`M`", where $n > 0$. Strings of the former type indicate n consecutive `A` operations, and strings of the latter type indicate n consecutive `M` operations.

Follow the format of the sample output.

Sample input	Output for the Sample Input
1 2 2 3 10 20 1 3 2 3 22 33 3 2 2 3 4 5 5 3 2 3 2 3 0 0 0 0 0 0	Case 1: 1A 2M Case 2: 1M 2A 1M Case 3: impossible Case 4: empty

Problem J
Pyramids
Problem ID: pyramids

It is not too hard to build a pyramid if you have a lot of identical cubes. On a flat foundation you lay, say, 10×10 cubes in a square. Centered on top of that square you lay a 9×9 square of cubes. Continuing this way you end up with a single cube, which is the top of the pyramid. The height of such a pyramid equals the length of its base, which in this case is 10. We call this a *high* pyramid.

If you think that a high pyramid is too steep, you can proceed as follows. On the 10×10 base square, lay an 8×8 square, then a 6×6 square, and so on, ending with a 2×2 top square (if you start with a base of odd length, you end up with a single cube on top, of course). The height of this pyramid is about half the length of its base. We call this a *low* pyramid.

Once upon a time (quite a long time ago, actually) there was a pharaoh who inherited a large number of stone cubes from his father. He ordered his architect to use all of these cubes to build a pyramid, not leaving a single one unused. The architect kindly explained that not every number of cubes can form a pyramid. With 10 cubes you can build a low pyramid with base 3. With 5 cubes you can build a high pyramid of base 2. But no pyramid can be built using exactly 7 cubes.

The pharaoh was not amused, but after some thinking he came up with new restrictions.

1. All cubes must be used.
2. You may build more than one pyramid, but you must build as few pyramids as possible.
3. All pyramids must be different.
4. Each pyramid must have a height of at least 2.
5. Satisfying the above, the largest of the pyramids must be as large as possible (i.e., containing the most cubes).
6. Satisfying the above, the next-to-largest pyramid must be as large as possible.
7. And so on...

Drawing figures and pictures in the sand, it took the architect quite some time to come up with the best solution.

Write a program that determines how to meet the restrictions of the pharaoh, given the number of cubes.

Input

The input consists of several test cases, each one on a single line. A test case is an integer c, where $1 \leq c \leq 10^6$, giving the number of cubes available.

The last test case is followed by a line containing a single zero.

Output

For each test case, display its case number followed by the pyramids to be built. The pyramids should be ordered with the largest first. Pyramids are specified by the length of their base followed by an `L` for low pyramids or an `H` for high pyramids. If two differenct pyramids have the same number of cubes, list the high pyramid first. Print "`impossible`" if it is not possible to meet the requirements of the pharaoh.

Follow the format of the sample output.

Sample input	Output for the Sample Input
29 28 0	Case 1: 3H 3L 2H Case 2: impossible