Homework Assignment #9

Note

This assignment is due 2:10PM Tuesday, June 7, 2011. Please write or type your answers on A4 (or similar size) paper. Drop your homework by the due time in Yih-Kuen Tsay's mail box on the first floor of Management College Building II. Late submission will be penalized by 20% for each working day overdue. You may discuss the problems with others, but copying answers is strictly forbidden.

There are five problems in this assignment, each accounting for 20 points.

Problems

1. (7.6) Consider Algorithm *Single_Source_Shortest_Paths* (discussed in class). Prove that the subgraph consisting of all the edges that belong to shortest paths from v, found during the execution of the algorithm, is a tree rooted at v.

2. (7.12)

   (a) Give an example of a weighted connected undirected graph $G = (V, E)$ and a vertex v, such that the minimum-cost spanning tree of G is the same as the shortest-path tree rooted at v.

   (b) Give an example of a weighted connected undirected graph $G = (V, E)$ and a vertex v, such that the minimum-cost spanning tree of G is very different from the shortest path tree rooted at v. Can the two trees be completely disjoint?

3. (7.16 modified)

   (a) Run the strongly connected components algorithm on the directed graph shown in Figure 1. When traversing the graph, the algorithm should follow the given DFS numbers. Show the *High* values as computed by the algorithm in each step.

   (b) Add the edge $(4, 2)$ to the graph and discuss the changes this makes to the execution of the algorithm.

4. (7.61) Let $G = (V, E)$ be a connected weighted undirected graph and T be a minimum-cost spanning tree (MCST) of G. Suppose that the cost of one edge $\{u, v\}$ in G is changed (*increased* or *decreased*); $\{u, v\}$ may or may not belong to T. Design an algorithm to either find a new MCST or to determine that T is still an MCST. The more efficient your algorithm is, the more points you will be credited for this problem. Explain why your algorithm is correct and analyze its time complexity.

5. (7.88) Let $G = (V, E)$ be a directed graph, and let T be a DFS tree of G. Prove that the intersection of the edges of T with the edges of any strongly connected component of G form a subtree of T.

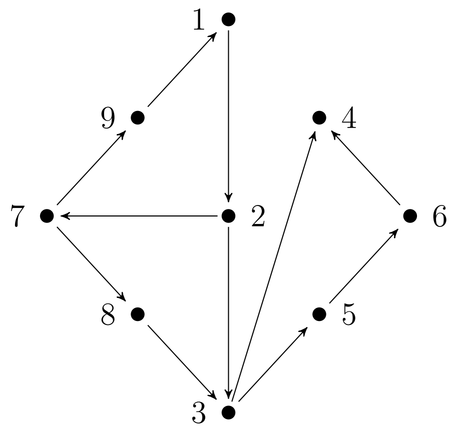

Figure 1: A directed graph with DFS numbers