Homework Assignment #3

Note

This assignment is due 2:10PM Monday, March 19, 2012. Please write or type your answers on A4 (or similar size) paper. Drop your homework by the due time in Yih-Kuen Tsay's mail box on the first floor of Management College Building II. Late submission will be penalized by 20% for each working day overdue. You may discuss the problems with others, but copying answers is strictly forbidden.

Problems

There are five problems in this assignment, each accounting for 20 points. (Note: problems marked with "(X.XX)" are taken from [Manber 1989] with probable adaptation.)

1. (3.4) Below is a theorem from Manber's book:

   For all constants $c > 0$ and $a > 1$, and for all monotonically increasing functions $f(n)$, we have $(f(n))^c = O(a^{f(n)})$.

   Prove, by using the above theorem, that for all constants $a, b > 0$, $(\log_2 n)^a = O(n^b)$.

2. (3.5) For each of the following pairs of functions, say whether $f(n) = O(g(n))$ and/or $f(n) = \Omega(g(n))$. Justify your answers.

	$f(n)$	$g(n)$
(a)	$(\log n)^{\log n}$	$\frac{n}{\log n}$
(b)	$\sqrt{n}$	$(\log n)^2$

3. (3.12) Solve the following recurrence relation:

   $T(n) = n + \sum_{i=1}^{n-1} T(i)$,

   where $T(1) = 1$.

4. Solve the following recurrence relation using *generating functions*. This is a very simple recurrence relation, but you must use generating functions in your solution.

$$\begin{cases} T(1) = 2 \\ T(2) = 4 \\ T(n) = 3T(n-1) - 2T(n-2), \quad n \geq 3 \end{cases}$$

5. (3.30) Use Equation 1, shown below, to prove that $S(n) = \sum_{i=1}^{n} \lceil \log_2(n/i) \rceil$ satisfies $S(n) = O(n)$.

Bounding a summation by an integral

If $f(x)$ is a monotonically increasing continuous function, then

$$\sum_{i=1}^{n} f(i) \leq \int_{x=1}^{x=n+1} f(x)dx. \qquad (1)$$