

Data Structures

A Supplement (Based on [Manber 1989])

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Heaps



- A (max) heap is a binary tree whose keys satisfy the heap property: the key of every node is greater than or equal to the key of any
- It supports the two basic operations of a priority queue:

of its children.

Heaps



- A (max) heap is a binary tree whose keys satisfy the heap property:
 - the key of every node is greater than or equal to the key of any of its children.
- 😚 It supports the two basic operations of a priority queue:
 - # Insert (x): insert the key x into the heap.
 - * Remove(): remove and return the largest key from the heap.

Heaps (cont.)



- A binary tree can be represented implicitly by an array A as follows:
 - 1. The root is stored in A[1].
 - 2. The left child of A[i] is stored in A[2i] and the right child is stored in A[2i+1].

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Heaps (cont.)



```
Algorithm Remove_Max_from_Heap (A, n);
begin
    if n = 0 then print "the heap is empty"
    else Top\_of\_the\_Heap := A[1];
        A[1] := A[n]; n := n - 1;
        parent := 1: child := 2:
        while child < n-1 do
              if A[child] < A[child + 1] then
                child := child + 1:
              if A[child] > A[parent] then
                swap(A[parent], A[child]);
                parent := child:
                child := 2 * child
              else child := n
```

end

Heaps (cont.)



```
Algorithm Insert_to_Heap (A, n, x);
begin
        n := n + 1:
       A[n] := x;
        child := n:
        parent := n div 2;
        while parent > 1 do
              if A[parent] < A[child] then
                swap(A[parent], A[child]);
                 child := parent;
                 parent := parent div 2
              else parent := 0
```

end

AVL Trees



Definition

An AVL tree is a binary search tree such that, for every node, the difference between the heights of its left and right subtrees is at most 1 (the height of an empty tree is defined as 0).

This definition guarantees a maximal height of $O(\log n)$ for any AVL tree of n nodes.

AVL Trees (cont.)



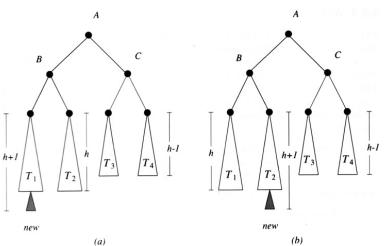


Figure 4.13 Insertions that invalidate the AVL property.

AVL Trees (cont.)



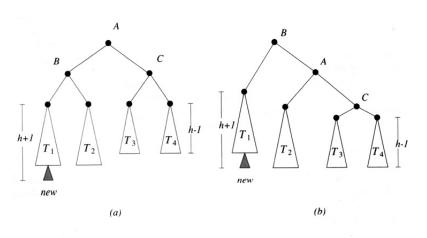


Figure 4.14 A single rotation: (a) Before. (b) After.

Source: [Manber 1989].

AVL Trees (cont.)



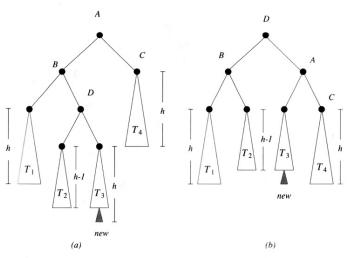


Figure 4.15 A double rotation: (a) Before. (b) After.

Union-Find



- There are n elements x_1, x_2, \dots, x_n divided into groups. Initially, each element is in a group by itself.
- Two operations on the elements and groups:
 - # find(A): returns the name of A's group.
 - union(A, B): combines A's and B's groups to form a new group with a unique name.
- To tell if two elements are in the same group, one may issue a find operation for each element and see if the returned names are the same.

Union-Find (cont.)



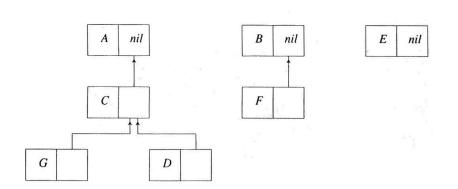


Figure 4.16 The representation for the union-find problem.

Source: [Manber 1989].

Balancing



- The root also stores the number of elements in (i.e., the size of) its group.
- To balance the tree resulted from a union operation, let the smaller group join the larger group and update the size of the larger group accordingly.

Theorem (Theorem 4.2)

If balancing is used, then any tree of height h must contain at least 2^h elements.

• Any sequence of m find or union operations (where $m \ge n$) takes $O(m \log n)$ steps.

Union-Find (cont.)



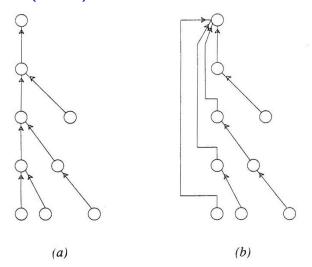


Figure 4.17 Path compression: (a) Before. (b) After.

Effect of Path Compression



Theorem (Theorem 4.3)

If both balancing and path compression are used, any sequence of m find or union operations (where $m \ge n$) takes $O(m \log^* n)$ steps.

The value of $\log^* n$ intuitively equals the number of times that one has to apply log to n to bring its value down to 1.

Code for Union-Find



```
Algorithm Union_Find_Init(A,n);
begin
  for i := 1 to n do
      A[i].parent := nil;
      A[i].size := 1
end
Algorithm Find(a);
begin
  if A[a].parent <> nil then
     A[a].parent := Find(A[a].parent);
     Find := A[a].parent;
  else
     Find := a
end
```

Code for Union-Find (cont.)



```
Algorithm Union(a,b);
begin
 x := Find(a);
  y := Find(b);
  if x \ll y then
     if A[x].size > A[y].size then
        A[v].parent := x;
        A[x].size := A[x].size + A[y].size;
     else
        A[x].parent := y;
        A[y].size := A[y].size + A[x].size
end
```