Homework Assignment #3

Note

This assignment is due 2:10PM Tuesday, March 21, 2017. Please write or type your answers on A4 (or similar size) paper. Drop your homework by the due time in Yih-Kuen Tsay's mail box on the first floor of Management College Building 2. Late submission will be penalized by 20% for each working day overdue. You may discuss the problems with others, but copying answers is strictly forbidden.

Problems

There are five problems in this assignment, each accounting for 20 points. (Note: problems marked with "(X.XX)" are taken from [Manber 1989] with probable adaptation.)

1. Consider the following algorithm for computing the square of a given non-negative integer.

   Algorithm mySquare(n);
   begin
       // assume that $n \geq 0$
       $x := n$;
       $y := 0$;
       while $x > 0$ **do**
           $y := y + 2x - 1$;
           $x := x - 1$;
       od
       ...
   end

   Let $Inv(n, x, y)$ denote the assertion:

   $$x \geq 0 \wedge y \geq 0 \wedge y = n^2 - x^2.$$

   Claim: $Inv(n, x, y)$ is a loop invariant of the while loop, assuming that $n \geq 0$.

   Prove the claim by induction. What does the loop invariant implicate when the while loop terminates?

2. (3.5) For each of the following pairs of functions, say whether $f(n) = O(g(n))$ and/or $f(n) = \Omega(g(n))$. Justify your answers.

	$f(n)$	$g(n)$
(a)	$\frac{n}{\log n}$	$(\log n)^2$
(b)	$n^4 2^n$	4^n

3. Solve the following recurrence relation using *generating functions*. This is a very simple recurrence relation, but for the purpose of practicing you must use generating functions in your solution.

   $$\begin{cases} T(1) = 1 \\ T(2) = 3 \\ T(n) = 2T(n-1) - T(n-2), \quad n \geq 3 \end{cases}$$

4. (3.26) Find the asymptotic behavior of the function $T(n)$ defined by the recurrence relation

$$T(n) = T(n/2) + \sqrt{n},\ T(1) = 1.$$

You can consider only values of n that are powers of 2.

5. (3.30) Use Equation 1, shown below, to prove that $S(n) = \sum_{i=1}^{n} \lceil \log_2(n/i) \rceil$ satisfies $S(n) = O(n)$.

Bounding a summation by an integral
If $f(x)$ is a monotonically increasing continuous function, then

$$\sum_{i=1}^{n} f(i) \leq \int_{x=1}^{x=n+1} f(x)dx. \tag{1}$$