Homework Assignment #7

Due Time/Date

2:10PM Tuesday, December 1, 2020. Late submission will be penalized by 20% for each working day overdue.

Note

Please write or type your answers on A4 (or similar size) paper. Drop your homework by the due time in Yih-Kuen Tsay's mail box on the first floor of Management College Building 2. You may discuss the problems with others, but copying answers is strictly forbidden.

Problems

There are five problems in this assignment, each accounting for 20 points.

1. (7.23) Describe an efficient implementation of the algorithm discussed in class for finding an Eulerian circuit in a graph. The algorithm should run in linear time and space. (Hint: try to interweave the discovery of a cycle and that of the separate Eulerian circuits in the connected components with the cycle removed in the induction step.)

2. (7.28) A **binary de Bruijn sequence** is a (cyclic) sequence of 2^n bits $a_1a_2\cdots a_{2^n}$ such that each binary string s of size n is represented somewhere in the sequence; that is, there exists a unique index i such that $s = a_ia_{i+1}\cdots a_{i+n-1}$ (where the indices are taken modulo 2^n). For example, the sequence 11010001 is a binary de Bruijn sequence for $n = 3$. Let $G_n = (V, E)$ be a directed graph defined as follows. The vertex set V corresponds to the set of all binary strings of size $n-1$ ($|V| = 2^{n-1}$). A vertex corresponding to the string $a_1a_2\cdots a_{n-1}$ has an edge leading to a vertex corresponding to the string $b_1b_2\cdots b_{n-1}$ if and only if $a_2a_3\cdots a_{n-1} = b_1b_2\cdots b_{n-2}$. Prove that G_n is a directed Eulerian graph, and discuss the implications for de Bruijn sequences.

3. (7.1) Consider the problem of finding balance factors in binary trees discussed in class (see slides for "Design by Induction"). Solve this problem using DFS. You need only to define preWORK and postWORK.

4. (7.3) Given as input a connected undirected graph G, a spanning tree T of G, and a vertex v, design an algorithm to determine whether T is a valid DFS tree of G rooted at v. In other words, determine whether T can be the output of DFS under some order of the edges starting with v. The running time of the algorithm should be $O(|V| + |E|)$.

5. Consider BFS of a directed graph $G = (V, E)$. If an edge (v, w) in E does not belong to the BFS tree and w is on a larger level, then the level numbers of w and v differ by at most 1.