Homework Assignment #8

Due Time/Date

2:20PM Tuesday, November 29, 2022. Late submission will be penalized by 20% for each working day overdue.

How to Submit

Please use a word processor or scan hand-written answers to produce a single PDF file. Name your file according to this pattern: "b107050xx-hw8". Upload the PDF file to the NTU COOL site for Algorithms 2022. You may discuss the problems with others, but copying answers is strictly forbidden.

Problems

There are five problems in this assignment, each accounting for 20 points. (Note: problems marked with "(X.XX)" are taken from [Manber 1989] with probable adaptation.)

1. Consider Dijkstra's algorithm for single-source shortest paths. The values of SP for all vertices may be stored in either an ordinary array or a min heap. How do these two implementations compare in terms of time complexity? Please explain.

2. (7.9) Prove that if the costs of all edges in a given connected graph are distinct, then the graph has exactly one unique minimum-cost spanning tree.

3. (7.12)

   (a) Give an example of a weighted connected undirected graph $G = (V, E)$ and a vertex v, such that the minimum-cost spanning tree of G is the same as the shortest-path tree rooted at v.

   (b) Give an example of a weighted connected undirected graph $G = (V, E)$ and a vertex v, such that the minimum-cost spanning tree of G is very different from the shortest path tree rooted at v. Can the two trees be completely disjoint?

4. The well-known Kruskal's algorithm computes the minimum-cost spanning tree of a given connected weighted undirected graph with n vertices as follows:

   Initially, it treats the n vertices as a forest of n trees, each of a single node. It then examines the edges one by one in increasing order of their weights. If the edge under examination connects two different trees (i.e., the edge does not complete a cycle), it is included in the forest (causing the forest to evolve).

   Please present the algorithm in suitable pseudocode utilizing the two operations of the Union-Find data structure. What is the time complexity of the algorithm? Please explain.

5. (7.61) Let $G = (V, E)$ be a connected weighted undirected graph and T be a minimum-cost spanning tree (MCST) of G. Suppose that the cost of one edge $\{u, v\}$ in G is *decreased*; $\{u, v\}$ may or may not belong to T. Design an algorithm to either find a new MCST or to determine that T is still an MCST. The more efficient your algorithm is, the more points you will be credited for this problem. Explain why your algorithm is correct and analyze its time complexity.