Advanced Encryption Standard (AES)

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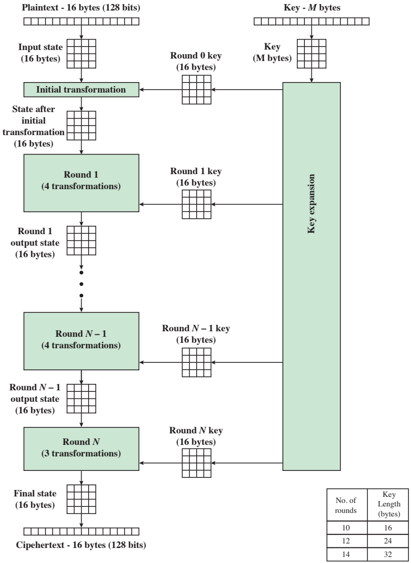
The Origin of AES

- A symmetric cipher intended to replace DES and 3DES (DES is slow and 3DES is three times as slow. Both use a 64-bit block size; a larger block size would be more secure.)
- A call for proposals for a new Advanced Encryption Standard issued in 1997 by NIST
- Selected algorithm: Rijndael, designed by Joan Daemen and Vincent Rijmen from Belgium.
- Published as FIPS PUB 197 in November, 2001
- Block size: 128 bits
- Key lengths: 128, 192, and 256 bits

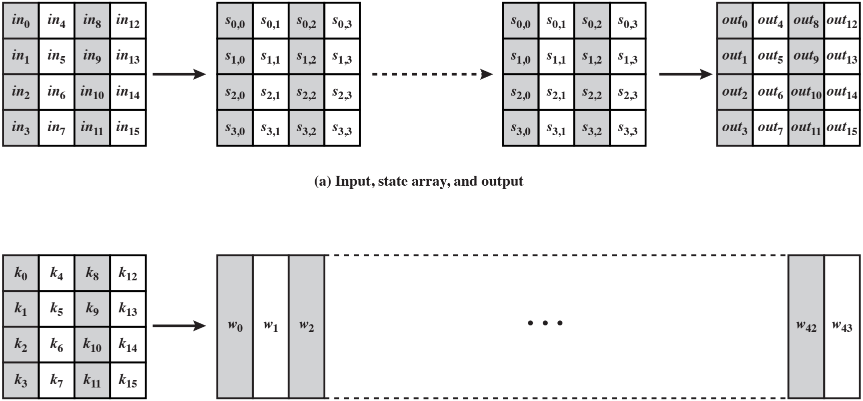
AES Encryption Process



No. of rounds	Key Length (bytes)
10	16
12	24
14	32

Source: Figure 5.1, Stallings 2010

AES Data Structures



in_0	in_4	in_8	in_{12}
in_1	in_5	in_9	in_{13}
in_2	in_6	in_{10}	in_{14}
in_3	in_7	in_{11}	in_{15}

$s_{0,0}$	$s_{0,1}$	$s_{0,2}$	$s_{0,3}$
$s_{1,0}$	$s_{1,1}$	$s_{1,2}$	$s_{1,3}$
$s_{2,0}$	$s_{2,1}$	$s_{2,2}$	$s_{2,3}$
$s_{3,0}$	$s_{3,1}$	$s_{3,2}$	$s_{3,3}$

$s_{0,0}$	$s_{0,1}$	$s_{0,2}$	$s_{0,3}$
$s_{1,0}$	$s_{1,1}$	$s_{1,2}$	$s_{1,3}$
$s_{2,0}$	$s_{2,1}$	$s_{2,2}$	$s_{2,3}$
$s_{3,0}$	$s_{3,1}$	$s_{3,2}$	$s_{3,3}$

out_0	out_4	out_8	out_{12}
out_1	out_5	out_9	out_{13}
out_2	out_6	out_{10}	out_{14}
out_3	out_7	out_{11}	out_{15}

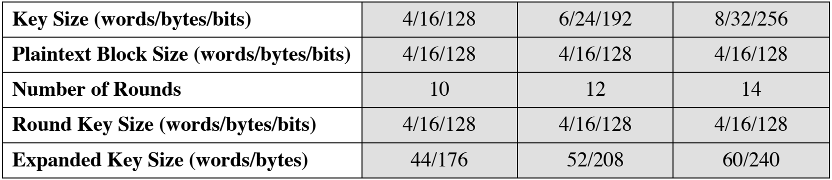
(a) Input, state array, and output

k_0	k_4	k_8	k_{12}
k_1	k_5	k_9	k_{13}
k_2	k_6	k_{10}	k_{14}
k_3	k_7	k_{11}	k_{15}

(b) Key and expanded key

Source: Figure 5.2, Stallings 2010

AES Parameters



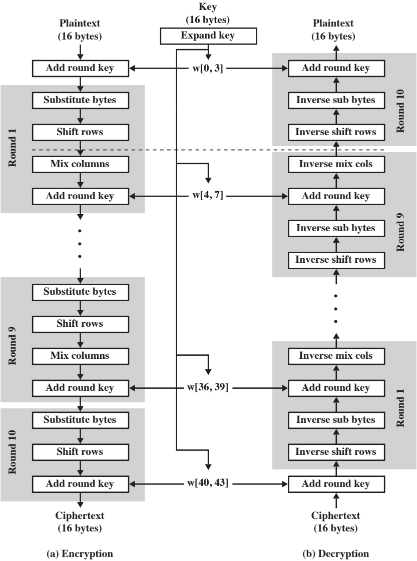
Key Size (words/bytes/bits)	4/16/128	6/24/192	8/32/256
Plaintext Block Size (words/bytes/bits)	4/16/128	4/16/128	4/16/128
Number of Rounds	10	12	14
Round Key Size (words/bytes/bits)	4/16/128	4/16/128	4/16/128
Expanded Key Size (words/bytes)	44/176	52/208	60/240

Source: Table 5.1, Stallings 2010

About the AES Cipher

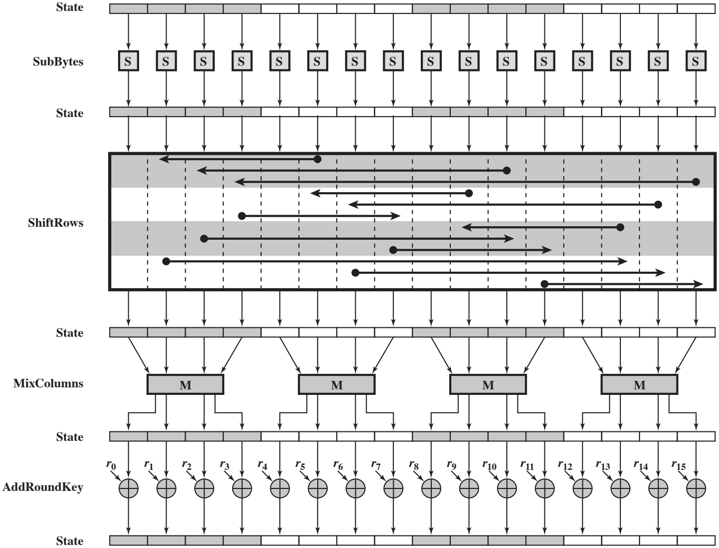
- Not a Feistel structure; entire data block processed in each round
- Input key expanded into 11 round keys of the same length
- Four stages used: **Substitute bytes**, **Shift rows** (the only permutation), **Mix columns**, **Add round key**
- Decryption algorithm different from encryption algorithm
- Correctness easy to verify.

AES Encryption and Decryption



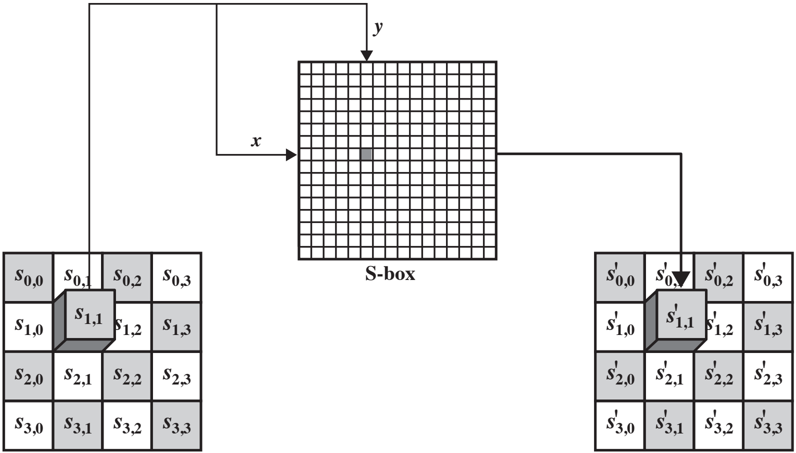
Source: Figure 5.3, Stallings 2010

AES Encryption Round



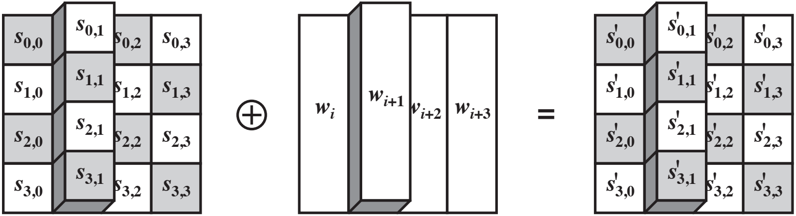
Source: Figure 5.4, Stallings 2010

AES Byte-Level Operations


y
x
S-box
$s_{0,0}$ $s_{0,1}$ $s_{0,2}$ $s_{0,3}$
$s_{1,0}$ $s_{1,1}$ $s_{1,2}$ $s_{1,3}$
$s_{2,0}$ $s_{2,1}$ $s_{2,2}$ $s_{2,3}$
$s_{3,0}$ $s_{3,1}$ $s_{3,2}$ $s_{3,3}$
$s'_{0,0}$ $s'_{0,1}$ $s'_{0,2}$ $s'_{0,3}$
$s'_{1,0}$ $s'_{1,1}$ $s'_{1,2}$ $s'_{1,3}$
$s'_{2,0}$ $s'_{2,1}$ $s'_{2,2}$ $s'_{2,3}$
$s'_{3,0}$ $s'_{3,1}$ $s'_{3,2}$ $s'_{3,3}$

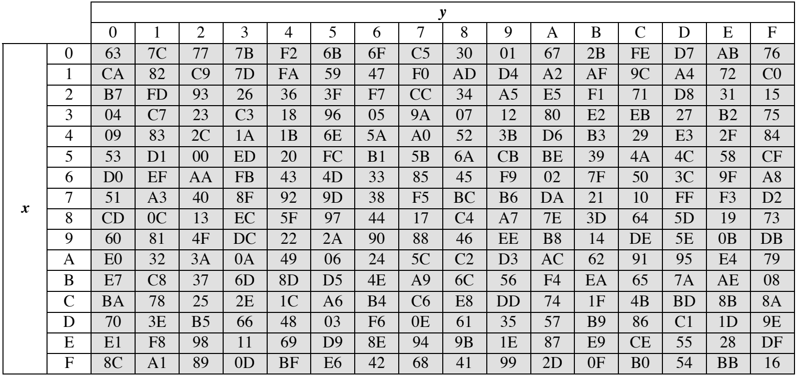
Source: Figure 5.5(a), Stallings 2010

AES Byte-Level Operations (cont.)



Source: Figure 5.5(b), Stallings 2010

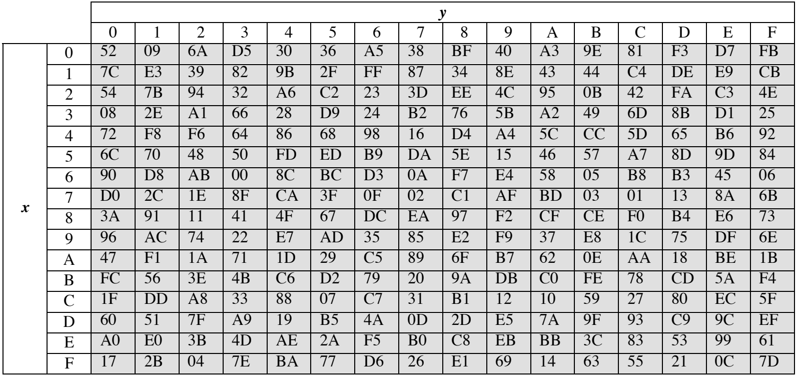
AES S-Boxes



		y															
		0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
x	0	63	7C	77	7B	F2	6B	6F	C5	30	01	67	2B	FE	D7	AB	76
	1	CA	82	C9	7D	FA	59	47	F0	AD	D4	A2	AF	9C	A4	72	C0
	2	B7	FD	93	26	36	3F	F7	CC	34	A5	E5	F1	71	D8	31	15
	3	04	C7	23	C3	18	96	05	9A	07	12	80	E2	EB	27	B2	75
	4	09	83	2C	1A	1B	6E	5A	A0	52	3B	D6	B3	29	E3	2F	84
	5	53	D1	00	ED	20	FC	B1	5B	6A	CB	BE	39	4A	4C	58	CF
	6	D0	EF	AA	FB	43	4D	33	85	45	F9	02	7F	50	3C	9F	A8
	7	51	A3	40	8F	92	9D	38	F5	BC	B6	DA	21	10	FF	F3	D2
	8	CD	0C	13	EC	5F	97	44	17	C4	A7	7E	3D	64	5D	19	73
	9	60	81	4F	DC	22	2A	90	88	46	EE	B8	14	DE	5E	0B	DB
	A	E0	32	3A	0A	49	06	24	5C	C2	D3	AC	62	91	95	E4	79
	B	E7	C8	37	6D	8D	D5	4E	A9	6C	56	F4	EA	65	7A	AE	08
	C	BA	78	25	2E	1C	A6	B4	C6	E8	DD	74	1F	4B	BD	8B	8A
	D	70	3E	B5	66	48	03	F6	0E	61	35	57	B9	86	C1	1D	9E
	E	E1	F8	98	11	69	D9	8E	94	9B	1E	87	E9	CE	55	28	DF
	F	8C	A1	89	0D	BF	E6	42	68	41	99	2D	0F	B0	54	BB	16

Source: Table 5.2, Stallings 2010

AES Inverse S-Boxes



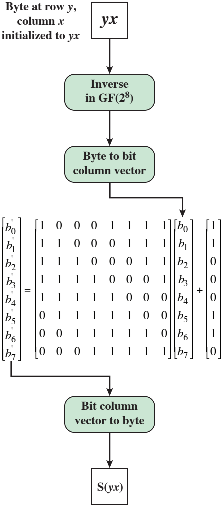
		y															
		0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
x	0	52	09	6A	D5	30	36	A5	38	BF	40	A3	9E	81	F3	D7	FB
	1	7C	E3	39	82	9B	2F	FF	87	34	8E	43	44	C4	DE	E9	CB
	2	54	7B	94	32	A6	C2	23	3D	EE	4C	95	0B	42	FA	C3	4E
	3	08	2E	A1	66	28	D9	24	B2	76	5B	A2	49	6D	8B	D1	25
	4	72	F8	F6	64	86	68	98	16	D4	A4	5C	CC	5D	65	B6	92
	5	6C	70	48	50	FD	ED	B9	DA	5E	15	46	57	A7	8D	9D	84
	6	90	D8	AB	00	8C	BC	D3	0A	F7	E4	58	05	B8	B3	45	06
	7	D0	2C	1E	8F	CA	3F	0F	02	C1	AF	BD	03	01	13	8A	6B
	8	3A	91	11	41	4F	67	DC	EA	97	F2	CF	CE	F0	B4	E6	73
	9	96	AC	74	22	E7	AD	35	85	E2	F9	37	E8	1C	75	DF	6E
	A	47	F1	1A	71	1D	29	C5	89	6F	B7	62	0E	AA	18	BE	1B
	B	FC	56	3E	4B	C6	D2	79	20	9A	DB	C0	FE	78	CD	5A	F4
	C	1F	DD	A8	33	88	07	C7	31	B1	12	10	59	27	80	EC	5F
	D	60	51	7F	A9	19	B5	4A	0D	2D	E5	7A	9F	93	C9	9C	EF
	E	A0	E0	3B	4D	AE	2A	F5	B0	C8	EB	BB	3C	83	53	99	61
	F	17	2B	04	7E	BA	77	D6	26	E1	69	14	63	55	21	0C	7D

Source: Table 5.2, Stallings 2010

An Example of SubBytes

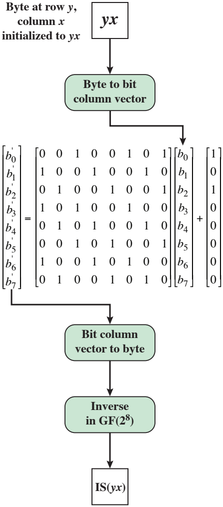
$$\begin{array}{|c|c|c|c|} \hline EA & 04 & 65 & 85 \\ \hline 83 & 45 & 5D & 96 \\ \hline 5C & 33 & 98 & B0 \\ \hline F0 & 2D & AD & C5 \\ \hline \end{array} \longrightarrow \begin{array}{|c|c|c|c|} \hline 87 & F2 & 4D & 97 \\ \hline EC & 6E & 4C & 90 \\ \hline 4A & C3 & 46 & E7 \\ \hline 8C & D8 & 95 & A6 \\ \hline \end{array}$$

Construction of S-Box and IS-Box


Byte at row *y*, column *x* initialized to *yx*
yx
Inverse in GF(2^8)
Byte to bit column vector
Bit column vector to byte
S(*yx*)

$$\begin{bmatrix} b_0' \\ b_1' \\ b_2' \\ b_3' \\ b_4' \\ b_5' \\ b_6' \\ b_7' \end{bmatrix} = \begin{bmatrix} 1&0&0&0&1&1&1&1 \\ 1&1&0&0&0&1&1&1 \\ 1&1&1&0&0&0&1&1 \\ 1&1&1&1&0&0&0&1 \\ 1&1&1&1&1&0&0&0 \\ 0&1&1&1&1&1&0&0 \\ 0&0&1&1&1&1&1&0 \\ 0&0&0&1&1&1&1&1 \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \\ b_7 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

(a) Calculation of byte at row y, column x of S-box


Byte at row *y*, column *x* initialized to *yx*
yx
Byte to bit column vector
Bit column vector to byte
Inverse in GF(2^8)
IS(*yx*)

$$\begin{bmatrix} b_0' \\ b_1' \\ b_2' \\ b_3' \\ b_4' \\ b_5' \\ b_6' \\ b_7' \end{bmatrix} = \begin{bmatrix} 0&0&1&0&0&1&0&1 \\ 1&0&0&1&0&0&1&0 \\ 0&1&0&0&1&0&0&1 \\ 1&0&1&0&0&1&0&0 \\ 0&1&0&1&0&0&1&0 \\ 0&0&1&0&1&0&0&1 \\ 1&0&0&1&0&1&0&0 \\ 0&1&0&0&1&0&1&0 \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \\ b_7 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

(a) Calculation of byte at row y, column x of IS-box

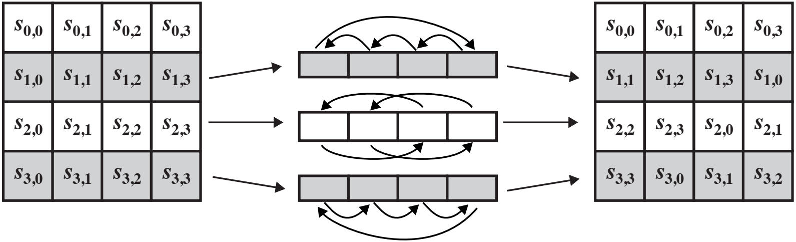
Source: Figure 5.6, Stallings 2010

Construction of the S-Box

- Initialization: 1st row: $\{00\}$, $\{01\}$, $\{02\}$, $\cdots$, $\{0F\}$; 2nd row: $\{10\}$, $\{11\}$, $\{12\}$, $\cdots$, $\{1F\}$; etc.
- Replace each byte with its multiplicative inverse; the value $\{00\}$ is mapped to itself.
- Apply the following (invertible) transformation:

$$\begin{bmatrix} b'_0 \\ b'_1 \\ b'_2 \\ b'_3 \\ b'_4 \\ b'_5 \\ b'_6 \\ b'_7 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \\ b_7 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

Shift Rows



$s_{0,0}$	$s_{0,1}$	$s_{0,2}$	$s_{0,3}$
$s_{1,0}$	$s_{1,1}$	$s_{1,2}$	$s_{1,3}$
$s_{2,0}$	$s_{2,1}$	$s_{2,2}$	$s_{2,3}$
$s_{3,0}$	$s_{3,1}$	$s_{3,2}$	$s_{3,3}$

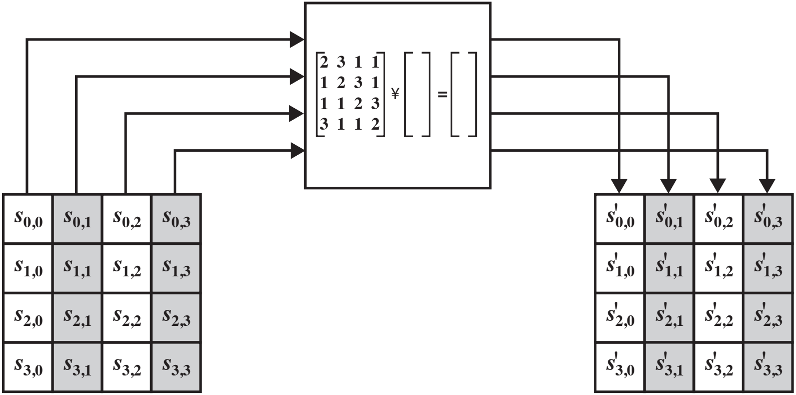
$s_{0,0}$	$s_{0,1}$	$s_{0,2}$	$s_{0,3}$
$s_{1,1}$	$s_{1,2}$	$s_{1,3}$	$s_{1,0}$
$s_{2,2}$	$s_{2,3}$	$s_{2,0}$	$s_{2,1}$
$s_{3,3}$	$s_{3,0}$	$s_{3,1}$	$s_{3,2}$

Source: Figure 5.7(a), Stallings 2010

An Example of `ShiftRows`

$$\begin{array}{|c|c|c|c|}\hline 87 & F2 & 4D & 97 \\ \hline EC & 6E & 4C & 90 \\ \hline 4A & C3 & 46 & E7 \\ \hline 8C & D8 & 95 & A6 \\ \hline \end{array} \longrightarrow \begin{array}{|c|c|c|c|}\hline 87 & F2 & 4D & 97 \\ \hline 6E & 4C & 90 & EC \\ \hline 46 & E7 & 4A & C3 \\ \hline A6 & 8C & D8 & 95 \\ \hline \end{array}$$

Mix Columns


$\begin{bmatrix} 2 & 3 & 1 & 1 \\ 1 & 2 & 3 & 1 \\ 1 & 1 & 2 & 3 \\ 3 & 1 & 1 & 2 \end{bmatrix} ¥ \begin{bmatrix} \\ \\ \\ \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \end{bmatrix}$
$s_{0,0}$ $s_{0,1}$ $s_{0,2}$ $s_{0,3}$
$s_{1,0}$ $s_{1,1}$ $s_{1,2}$ $s_{1,3}$
$s_{2,0}$ $s_{2,1}$ $s_{2,2}$ $s_{2,3}$
$s_{3,0}$ $s_{3,1}$ $s_{3,2}$ $s_{3,3}$
$s'_{0,0}$ $s'_{0,1}$ $s'_{0,2}$ $s'_{0,3}$
$s'_{1,0}$ $s'_{1,1}$ $s'_{1,2}$ $s'_{1,3}$
$s'_{2,0}$ $s'_{2,1}$ $s'_{2,2}$ $s'_{2,3}$
$s'_{3,0}$ $s'_{3,1}$ $s'_{3,2}$ $s'_{3,3}$

Source: Figure 5.7(b), Stallings 2010

An Example of `MixColumns`

$$
\begin{array}{|c|c|c|c|}
\hline
87 & F2 & 4D & 97 \\ \hline
6E & 4C & 90 & EC \\ \hline
46 & E7 & 4A & C3 \\ \hline
A6 & 8C & D8 & 95 \\ \hline
\end{array}
\longrightarrow
\begin{array}{|c|c|c|c|}
\hline
47 & 40 & A3 & 4C \\ \hline
37 & D4 & 70 & 9F \\ \hline
94 & E4 & 3A & 42 \\ \hline
ED & A5 & A6 & BC \\ \hline
\end{array}
$$

$$
\begin{array}{lcl}
(\{02\} \bullet \{87\}) & = & 00010101 \\
(\{03\} \bullet \{6E\}) & = & 10110010 \\
\{46\} & = & 01000110 \\
\{A6\} & = & 10100110 \\
\hline
 & & 01000111 \ (= \{47\})
\end{array}
$$

InvMixColumns

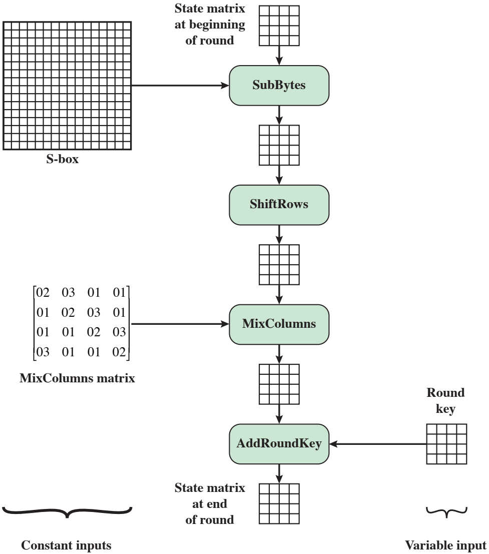
$$\begin{bmatrix} 0E & 0B & 0D & 09 \\ 09 & 0E & 0B & 0D \\ 0D & 09 & 0E & 0B \\ 0B & 0D & 09 & 0E \end{bmatrix} \begin{bmatrix} 02 & 03 & 01 & 01 \\ 01 & 02 & 03 & 01 \\ 01 & 01 & 02 & 03 \\ 03 & 01 & 01 & 02 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

An Example of `AddRoundKey`

$$\begin{array}{|c|c|c|c|}\hline 47 & 40 & A3 & 4C \\ \hline 37 & D4 & 70 & 9F \\ \hline 94 & E4 & 3A & 42 \\ \hline ED & A5 & A6 & BC \\ \hline \end{array} \oplus \begin{array}{|c|c|c|c|}\hline AC & 19 & 28 & 57 \\ \hline 77 & FA & D1 & 5C \\ \hline 66 & DC & 29 & 00 \\ \hline F3 & 21 & 41 & 6A \\ \hline \end{array}$$

$$= \begin{array}{|c|c|c|c|}\hline EB & 59 & 8B & 1B \\ \hline 40 & 2E & A1 & C3 \\ \hline F2 & 38 & 13 & 42 \\ \hline 1E & 84 & E7 & D2 \\ \hline \end{array}$$

Inputs for Single AES Round



State matrix at beginning of round

S-box

SubBytes

ShiftRows

$$\begin{bmatrix} 02 & 03 & 01 & 01 \\ 01 & 02 & 03 & 01 \\ 01 & 01 & 02 & 03 \\ 03 & 01 & 01 & 02 \end{bmatrix}$$

MixColumns matrix

MixColumns

Round key

AddRoundKey

State matrix at end of round

Constant inputs

Variable input

Source: Figure 5.8, Stallings 2010

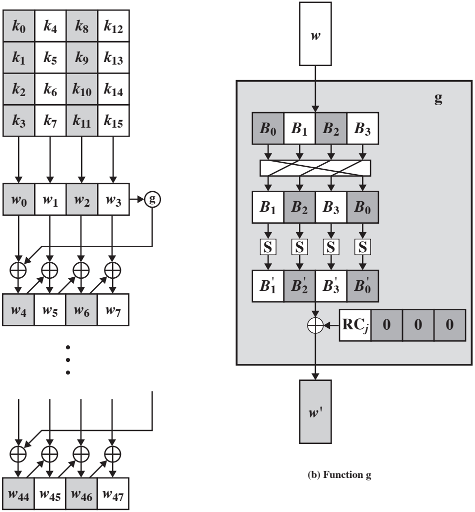
Key Expansion

```
KeyExpansion (byte key[16],word w[44])
{
    word temp
    for ($i = 0$; $i < 4$; $i++$)
        w[i] $=$ (key[$4*i$],key[$4*i+1$],key[$4*i+2$],key[$4*i+3$]);
    for ($i = 4$; $i < 44$; $i++$)
    {   temp $=$ w[$i-1$];
        if ($i \bmod 4 = 0$) temp $=$ SubWord(RotWord(temp))$\oplus$Rcon[$i/4$];
        w[i] $=$ w[$i-4$]$\oplus$temp
    }
}
```

Rcon[j] $=$ (RC[j],0,0,0), with RC[1]=1, RC[j]=2$\bullet$RC[$j-1$]

j	1	2	3	4	5	6	7	8	9	10
$RC[j]$	01	02	04	08	10	20	40	80	$1B$	36

AES Key Expansion


k_0 k_4 k_8 k_{12}
k_1 k_5 k_9 k_{13}
k_2 k_6 k_{10} k_{14}
k_3 k_7 k_{11} k_{15}
w_0 w_1 w_2 w_3
g
w_4 w_5 w_6 w_7
w_{44} w_{45} w_{46} w_{47}
w
g
B_0 B_1 B_2 B_3
B_1 B_2 B_3 B_0
S S S S
B_1' B_2' B_3' B_0'
RC_j 0 0 0
w'

(a) Overall algorithm

(b) Function g

Source: Figure 5.9, Stallings 2010

An Example of Key Expansion

Suppose the round key (Words 32, 33, 34, and 35) for Round 8 is

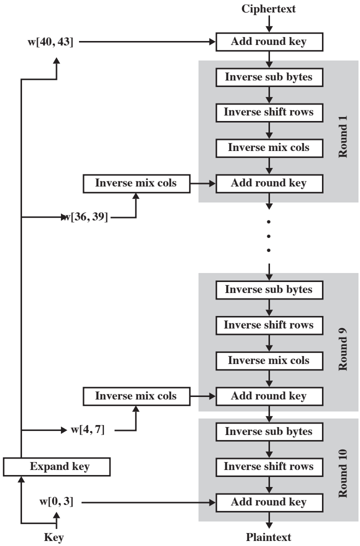
EA D2 73 21 B5 8D BA D2 31 2B F5 60 7F 8D 29 2F.

The first 4 bytes (Word 36) of the round key for round 9 are calculated as follows:

i	temp	RotWord	SubWord	Rcon(9)
36	7F8D292F	8D292F7F	5DA515D2	1B000000

XOR	w[$i-4$]	w[i]
46A515D2	EAD27321	AC7766F3

Equivalent Inverse Cipher



Source: Figure 5.10, Stallings 2010

Equivalent Inverse Cipher (cont.)

- Interchanging **InvShiftRows** and **InvSubBytes**:

  $$\mathrm{InvShiftRows}[\mathrm{InvSubBytes}(S_i)] = \mathrm{InvSubBytes}[\mathrm{InvShiftRows}(S_i)]$$

- Interchanging **AddRoundKey** and **InvMixColumns**:
  For a given state S_i and a given round key w_j,

  $$\mathrm{InvMixColumns}(S_i \oplus w_j)$$
  $$= [\mathrm{InvMixColumns}(S_i)] \oplus [\mathrm{InvMixColumns}(w_j)]$$

Implementation in 32-Bit Processes

$$\begin{bmatrix} e_{0,j} \\ e_{1,j} \\ e_{2,j} \\ e_{3,j} \end{bmatrix} = \begin{bmatrix} 02 & 03 & 01 & 01 \\ 01 & 02 & 03 & 01 \\ 01 & 01 & 02 & 03 \\ 03 & 01 & 01 & 02 \end{bmatrix} \begin{bmatrix} S[a_{0,j}] \\ S[a_{1,j+1}] \\ S[a_{2,j+2}] \\ S[a_{3,j+3}] \end{bmatrix} \oplus \begin{bmatrix} k_{0,j} \\ k_{1,j} \\ k_{2,j} \\ k_{3,j} \end{bmatrix} =$$

$$\left(\begin{bmatrix} 02 \\ 01 \\ 01 \\ 03 \end{bmatrix} \bullet S[a_{0,j}] \right) \oplus \left(\begin{bmatrix} 03 \\ 02 \\ 01 \\ 01 \end{bmatrix} \bullet S[a_{1,j+1}] \right) \oplus \left(\begin{bmatrix} 01 \\ 03 \\ 02 \\ 01 \end{bmatrix} \bullet S[a_{2,j+2}] \right) \oplus$$

$$\left(\begin{bmatrix} 01 \\ 01 \\ 03 \\ 02 \end{bmatrix} \bullet S[a_{3,j+3}] \right) \oplus \begin{bmatrix} k_{0,j} \\ k_{1,j} \\ k_{2,j} \\ k_{3,j} \end{bmatrix}$$

Implementation in 32-Bit Processes (cont.)

To facilitate the preceding calculation, four tables may be defined:

$$T_0(x) = \left(\begin{bmatrix} 02 \\ 01 \\ 01 \\ 03 \end{bmatrix} \bullet S[x] \right) ; \quad T_1(x) = \left(\begin{bmatrix} 03 \\ 02 \\ 01 \\ 01 \end{bmatrix} \bullet S[x] \right)$$

$$T_2(x) = \left(\begin{bmatrix} 01 \\ 03 \\ 02 \\ 01 \end{bmatrix} \bullet S[x] \right) ; \quad T_3(x) = \left(\begin{bmatrix} 01 \\ 01 \\ 03 \\ 02 \end{bmatrix} \bullet S[x] \right)$$

These tables can be pre-computed.