## Homework Assignment \#0

## Note

This assignment is due 9:10AM Thursday, September 24, 2009 (when the next class meeting starts). Please write or type your answers on A4 (or similar size) paper. Put your completed homework on the instructor's desk before the class starts. No late submission will be accepted. You may discuss the problems with others, but copying answers is strictly forbidden.

## Problem

Sudoku (數獨) is a logic-based, combinatorial number-placement puzzle. The objective is to fill a $9 \times 9$ grid so that each column, each row, and each of the nine $3 \times 3$ boxes (also called blocks or regions) contains the digits from 1 to 9 only one time each. The puzzle setter provides a partially completed grid. (from the Wikipedia page: http://en.wikipedia.org/wiki/Sudoku)

In this problem, we consider a mini version of the Sudoku puzzle with a $4 \times 4$ grid as below.

| 1 | 2 | 3 | 4 |
| :--- | :--- | :--- | :--- |
| 3 | 4 | 1 | 2 |
| 2 | 1 | 4 | 3 |
| 4 | 3 | 2 | 1 |

1. Write a propositional formula that characterizes all possible solutions to the following puzzle.

|  | 2 |  | 4 |
| :--- | :--- | ---: | ---: |
| 3 |  | 1 |  |
|  | 1 |  | 3 |
| 4 |  | 2 |  |

So, if you submit your formula to a satisfiability solver, it will return a truth assignment, if there is one, that satisfies the formula and hence gives a solution to the puzzle.
2. Given a solution as shown below (where $n_{i j}$ 's are numbers from the set $\{1,2,3,4\}$ ) to the preceding puzzle, write a propositional formula that characterizes all possible remaining solutions (which are different from the given one).

| $n_{11}$ | 2 | $n_{13}$ | 4 |
| :---: | :---: | :---: | :---: |
| 3 | $n_{21}$ | 1 | $n_{24}$ |
| $n_{31}$ | 1 | $n_{33}$ | 3 |
| 4 | $n_{42}$ | 2 | $n_{44}$ |

