Inference Rules of Hoare Logic

$$\{Q[E/x]\} \ x := E \ \{Q\}$$
 (Assignment)

Note: to treat multiple assignments, view x as a list of distinct variables and E as a list of expressions.

$$\overline{\{Q[(b;i:E)/b]\}\ b[i] := E\ \{Q\}}$$
 (Assignment: array)

$$\frac{\{P\}\ S_1\ \{Q\}\qquad \{Q\}\ S_2\ \{R\}}{\{P\}\ S_1; S_2\ \{R\}}$$
 (Sequence)

$$\frac{\{P \land B\} \ S_1 \ \{Q\} \qquad \{P \land \neg B\} \ S_2 \ \{Q\}}{\{P\} \ \text{if } B \ \text{then } S_1 \ \text{else } S_2 \ \text{fi} \ \{Q\}}$$
(Conditional)

"if B then S fi" can be treated as "if B then S else skip fi" or directly with the following rule:

$$\frac{\{P \land B\} \ S \ \{Q\} \qquad P \land \neg B \to Q}{\{P\} \ \textbf{if} \ B \ \textbf{then} \ S \ \textbf{fi} \ \{Q\}}$$
 (If-Then)

$$\frac{\{P \land B\} \ S \ \{P\}}{\{P\} \ \mathbf{while} \ B \ \mathbf{do} \ S \ \mathbf{od} \ \{P \land \neg B\}}$$
 (while)

$$\frac{P \to P' \qquad \{P'\} \ S \ \{Q'\} \qquad Q' \to Q}{\{P\} \ S \ \{Q\}}$$
 (Consequence)

$$\frac{\text{"proc p(in } x; \text{ in out } y; \text{ out } z); \{P\} \ S \ \{Q\}; \text{" is proved}}{\{P[a, b/x, y] \land I\}\} \ p(a, b, c) \ \{Q[b, c/y, z] \land I\}}$$
(Procedure Call)

where b, c are (lists of) distinct variables and I does not refer to variables changed by procedure p.

$$\frac{\{P \land B\} \ S \ \{P\} \qquad \{P \land B \land t = Z\} \ S \ \{t < Z\} \qquad P \land B \rightarrow (t \geq 0)}{\{P\} \ \textbf{while} \ B \ \textbf{do} \ S \ \textbf{od} \ \{P \land \neg B\}} \qquad \qquad (\textbf{while}: \ \text{simply total})$$

$$\frac{\{P \land B\} \ S \ \{P\} \qquad \{P \land B \land \delta = D\} \ S \ \{\delta \prec D\} \qquad P \land B \rightarrow (\delta \in W)}{\{P\} \ \textbf{while} \ B \ \textbf{do} \ S \ \textbf{od} \ \{P \land \neg B\}} \qquad (\textbf{while}: \ \textbf{well-founded})$$

Auxiliary Rules:

$$\frac{P \to P' \quad \{P'\} \ S \ \{Q\}}{\{P\} \ S \ \{Q\}}$$

$$\frac{\{P\} \ S \ \{Q'\} \qquad Q' \to Q}{\{P\} \ S \ \{Q\}}$$

$$\frac{\{P_1\} \ S \ \{Q_1\} \qquad \{P_2\} \ S \ \{Q_2\}}{\{P_1 \land P_2\} \ S \ \{Q_1 \land Q_2\}}$$

$$\frac{\{P_1\} \ S \ \{Q_1\} \qquad \{P_2\} \ S \ \{Q_2\}}{\{P_1 \lor P_2\} \ S \ \{Q_1 \lor Q_2\}}$$
(Conjunction)
$$\frac{\{P_1\} \ S \ \{Q_1\} \qquad \{P_2\} \ S \ \{Q_2\}}{\{P_1 \lor P_2\} \ S \ \{Q_1 \lor Q_2\}}$$