# Domain Modeling: Essence and Some Highlights <br> (Based partly on [Fowler 1997, Analysis Patterns]) 

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Introduction

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## What Is Domain Modeling?

Domain modeling is an activity of requirements/systems analysis for constructing a conceptual model, usually called the domain model, of the application/problem domain.
A domain model represents real-world entities/concepts and their relations, to help understand the problem and provide guidelines for software development.
The focus is often on the data part, though the behavioral aspect is inevitably considered in the modeling process.
Virtues to pursue: simplicity, flexibility, and reusability.

## Domain Models in UML

A conceptual/domain model may be described using various modeling notations such as UML class diagrams.
In a UML class diagram, concepts are represented by classes and relations by relationships, mostly associations and generalizations.
Note: you may want to review the lecture "UML: An Overview" to recall the basics of modeling and UML classes and relationships.

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- Multiplicity/cardinality is the most fundamental constraint to consider for a relationship.
Constraints that cannot be easily captured by multiplicities may be stated in a note. (Can you think of one such constraint?)


## Multiplicities of Relationships

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So, many-to-one and one-to-many relationships/relations should be treated differently.
- One should also be careful about from which side of a "one-to-one" relationship the relation is a total function.


## Sets and Types

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A multiset allows repetitions of a same element; use this notion when the ordinary set is not suitable.

- One can think of an element a from a set $A$ as being of type $A$.

So, types or data types basically are just sets; and subtypes are subsets. (More about this later.)

## Tuples and Records

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The Cartesian product, or simply product, of $A$ and $B$, written as $A \times B$, is the set of all pairs $(x, y)$ such that $x \in A$ and $y \in B$.

- For example, $\{a, b\} \times\{0,1,2\}=\{(a, 0),(a, 1),(a, 2),(b, 0),(b, 1),(b, 2)\}$.


## Tuples and Records (cont.)

Cartesian products generalize to $k$ sets, $A_{1}, A_{2}, \ldots, A_{k}$, written as $A_{1} \times A_{2} \times \ldots \times A_{k}$.
So, every element of $A_{1} \times A_{2} \times \ldots \times A_{k}$ is a $k$-tuple.
$A^{k}$ is a shorthand for $A \times A \times \ldots \times A$ ( $k$ times).

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- A record is essentially a generalization of a tuple, where every component is given a name, called a field name or attribute.
- Below is an example record: (ID: "IM5027", Title: "Software Development Methods", Credit: 3).


## Relations

A subset $R$ of $A_{1} \times A_{2} \times \ldots \times A_{k}$ is called a $k$-ary relation on $A_{1}, A_{2}, \ldots, A_{k}$.
We usually write $R\left(a_{1}, a_{2}, \ldots, a_{k}\right)$ to denote that $\left(a_{1}, a_{2}, \ldots, a_{k}\right) \in R$.
So, one can view a relation $R \subseteq A_{1} \times A_{2} \times \ldots \times A_{k}$ as a predicate.
When the $A_{i}$ 's are the same set $A$, it is simply called a $k$-ary relation on $A$.

## Relations (cont.)

A 1-ary relation is usually called a unary relation, which is also a way of defining subsets from an existing set.
A 2-ary relation is called a binary relation; for a binary relation $R, R(x, y)$ is also written as $x R y$.

- A binary relation $R \subseteq A \times B$ is said to be total if, for every $x \in A$, there exists some $y \in B$ such that $x R y$.
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$R \subseteq\left(A_{1} \times A_{2} \times \ldots \times A_{m}\right) \times\left(B_{1} \times B_{2} \times \ldots \times B_{n}\right)$ is a binary relation on $A_{1} \times A_{2} \times \ldots \times A_{m}$ and $B_{1} \times B_{2} \times \ldots \times B_{n}$.


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Alternatively, $R \subseteq A_{1} \times A_{2} \times \ldots \times A_{m} \times B_{1} \times B_{2} \times \ldots \times B_{n}$ is a $(m+n)$-ary relation.

## Functions

A (total) function (or mapping) $f$ from $D$ to $R$, denoted $f: D \longrightarrow R$, maps every element in $D$, called the domain of $f$, to some element in $R$, called the range of $f$.

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A function $f: D \longrightarrow R$ may be seen as a special kind of total binary relation $f \subseteq D \times R$ that is functional (one-to-one or many-to-one), i.e., for every $d \in D$, there is exactly an $r \in R$ s.t. $(d, r) \in f$, written usually as $f(d)=r$.


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A partial function may not produce an output for some inputs.

## Functions (cont.)

A function is said to be $k$-ary if its domain is a product of $k$ sets.
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Recall that $f$ may be seen as a special kind of binary relation, i.e., $f \subseteq\left(D_{1} \times D_{2} \times \ldots \times D_{k}\right) \times R$.

Function $f$ may also be seen as a special kind of $(k+1)$-ary relation, i.e., $f \subseteq D_{1} \times D_{2} \times \ldots \times D_{k} \times R$.

## "Multi-Valued Functions"

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- When a binary relation $f \subseteq D \times R$ is total but not functional, it is sometimes referred to as a "multi-valued function".Can we represent it as a real (single-valued) function?
Define a function $g: D \longrightarrow 2^{R}$ (from $D$ to the power set of $R$ ) such that, for every $d \in D, g(d)=\{r \in R \mid(d, r) \in f\}$.
- Function $g$ is single-valued and faithfully represents $f$.


## Subsets, Subtypes, and Subclasses

How can subtypes, or even subclasses, simply be viewed as subsets?
(Classes have their behavioral aspects, but that does not concern us in this lecture, which focuses on the data part).

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- Mathematical relations can be conveniently used to represent types/classes and are themselves sets.
A $k$-ary relation, when seen as a predicate, constrains its $k$ components and nothing beyond.
A $k$-tuple $\left(d_{1}, d_{2}, \ldots, d_{k}\right)$ in a $k$-ary relation may be extended as a $(k+1)$-tuple $\left.\left(d_{1}, d_{2}, \ldots, d_{k},\right)^{\prime}\right)$, where the $(k+1)$-th component may contain any value ("don't care"), denoted by ..
The extension may be generalized to include more than one additional components.


## Why Mathematics?

- It is precise.

Being abstract/conceptual does not imply being vague/imprecise.
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- It is common ultimately, for all.

It is expressive.

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Can you see an association relationship as a binary relation mathematically?

Can you see in an association two functions mathematically?

## An Association Class

| Project | assigned | members | Person |
| :---: | :---: | :---: | :---: |
|  | $*$ | $*$ |  |
|  |  |  |  |
|  |  |  |  |

A many-to-many relation (at the operational level) should be avoided. Why?

## An Association Class

| Project | assigned | members | Person |
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|  | $*$ | $*$ |  |
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A many-to-many relation (at the operational level) should be avoided. Why? It may instead be represented as follows.


The class ProjAssignment is called an association class, created to represent the original many-to-many association relation between Project and Person.

## The Abstract Concept/Class of "Party"



The Party generalization may apply to other entities, e.g., Post.

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The Party generalization may apply to other entities, e.g., Post.
Can you see Person and Organization as subsets of Party mathematically?

## The Party Abstraction Simplifies Relations

## 



## Hierarchies



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What if additional levels are needed?

## Hierarchies



What if additional levels are needed?
Modeling a hierarchy with explicit levels is inflexible.

## Hierarchies (cont.)

A hierarchical association provides better flexibility:


## Hierarchies (cont.)

A hierarchical association provides better flexibility:


Can you see the hierarchical association as a binary relation on Organization mathematically?

## More about Hierarchies

## What if several different hierarchies are needed?



This will become messy, when there are many hierarchies.

## Typed Relationship



## Accountability



## Knowledge vs. Operational Levels



Note: things at the operational level change on a daily basis, while those on the knowledge level change much less frequently.

## Quantity and Measurement



Note: this model is useful when there are too many possible measurements for a person. The types in PhenomenonType are what can be measured.

## Concluding Remarks

Domain modeling requires domain knowledge and experience.
Experience can be passed on and learned by good examples, namely patterns.
Patterns are not fixed and should be adapted to fit your needs.

- Always strive for simplicity, flexibility, and reusability.

