## Inference Rules of Hoare Logic

$$\{Q[E/x]\}\ x := E\ \{Q\}\tag{Assignment}$$

Note: to treat multiple assignments, view x as a list of distinct variables and E as a list of expressions.

$$\{Q[(b;i:E)/b]\}\ b[i] := E\ \{Q\}$$
(Assignment: array)

$$\{P\}$$
 skip  $\{P\}$  (Skip)

$$\frac{\{P\} S_1 \{Q\} \quad \{Q\} S_2 \{R\}}{\{P\} S_1; S_2 \{R\}}$$
(Sequence)

$$\frac{\{P \land B\} S_1 \{Q\}}{\{P\} \text{ if } B \text{ then } S_1 \text{ else } S_2 \text{ fi } \{Q\}}$$
(Conditional)

"if B then S fi" can be treated as "if B then S else skip fi" or directly with the following rule:

$$\frac{\{P \land B\} S \{Q\} \qquad P \land \neg B \to Q}{\{P\} \text{ if } B \text{ then } S \text{ fi } \{Q\}}$$
(If-Then)

$$\frac{\{P \land B\} S \{P\}}{\{P\} \text{ while } B \text{ do } S \text{ od } \{P \land \neg B\}}$$
(while)

$$\frac{P \to P' \quad \{P'\} \ S \ \{Q'\} \qquad Q' \to Q}{\{P\} \ S \ \{Q\}}$$
(Consequence)

"proc p(in x; in out y; out z);  $\{P\} S \{Q\}$ ;" is proved (Procedure Call)  $\{P[a, b/x, y] \land I\} p(a, b, c) \{Q[b, c/y, z] \land I\}$ 

where b, c are (lists of) distinct variables and I does not refer to variables changed by procedure p.

$$\frac{\{P \land B\} S \{P\} \quad \{P \land B \land t = Z\} S \{t < Z\} \quad P \land B \to (t \ge 0)}{\{P\} \text{ while } B \text{ do } S \text{ od } \{P \land \neg B\}}$$
(while: simply total)  
$$\frac{\{P \land B\} S \{P\} \quad \{P \land B \land \delta = D\} S \{\delta \prec D\} \quad P \land B \to (\delta \in W)}{\{P\} \text{ while } B \text{ do } S \text{ od } \{P \land \neg B\}}$$
(while: well-founded)

Auxiliary Rules:

$\frac{\{P'\} S \{Q\}}{S \{Q\}} $ (Strengtheni	ing Precondition)
$\frac{Q'}{S \{Q\}} \qquad (Weakening)$	ng Postcondition)
$egin{array}{llllllllllllllllllllllllllllllllllll$	(Conjunction)
$\begin{array}{l} Q_1 \} & \{P_2\} \ S \ \{Q_2\} \\ \hline P_2 \} \ S \ \{Q_1 \lor Q_2\} \end{array}$	(Disjunction)