Homework Assignment #1

Note

This assignment is due 9:10AM Wednesday, October 7, 2009. Please write or type your answers on A4 (or similar size) paper. Put your completed homework on the instructor's desk before the class starts. For late submissions, please drop them in Yih-Kuen Tsay's mail box on the first floor of Management Building II. Late submission will be penalized by 20% for each working day overdue. You may discuss the problems with others, but copying answers is strictly forbidden.

Problems

We assume the binding powers of the logical connectives and the entailment symbol decrease in this order: $\neg$, $\{\wedge, \vee\}$, $\rightarrow$, $\leftrightarrow$, $\vdash$.

1. Prove, using Gentzen's System LK, the validity of the following sequents:

   (a) $\vdash (\neg p \vee q) \rightarrow (p \rightarrow q)$ (10 points)

   (b) $p \vee q \rightarrow r \vdash (p \rightarrow r) \wedge (q \rightarrow r)$ (10 points)

   (c) $\vdash (p \rightarrow (q \rightarrow r)) \rightarrow (p \wedge q \rightarrow r)$ (10 points)

   (d) $\vdash ((p \rightarrow q) \rightarrow p) \rightarrow p$ (10 points)

2. We presented in class the $\wedge : Left_1$ and $\wedge : Left_2$ rules of Gentzen's System LK as follows.

$$\frac{\Gamma, A \vdash \Delta}{\Gamma, A \wedge B \vdash \Delta} \,(\wedge : Left_1) \qquad\qquad \frac{\Gamma, B \vdash \Delta}{\Gamma, A \wedge B \vdash \Delta} \,(\wedge : Left_2)$$

   We also suggested that, for convenience, the following $\wedge : Left$ rule can be used instead:

$$\frac{\Gamma, A, B \vdash \Delta}{\Gamma, A \wedge B \vdash \Delta} \,(\wedge : Left)$$

   Show that every proof using the $\wedge : Left_1$ or $\wedge : Left_2$ rules may be converted into one using the $\wedge : Left$ rule and vice versa. (20 points)

3. Prove, using *Natural Deduction*, the validity of the following sequents:

   (a) $\vdash (p \rightarrow q) \rightarrow (\neg p \vee q)$ (10 points)

   (b) $(p \rightarrow r) \wedge (q \rightarrow r) \vdash p \vee q \rightarrow r$ (10 points)

   (c) $\vdash (p \wedge q \rightarrow r) \rightarrow (p \rightarrow (q \rightarrow r))$ (10 points)

   (d) $\vdash ((p \rightarrow q) \rightarrow p) \rightarrow p$ (10 points)