

Natural Deduction in the Sequent Form

(for Classic Logic)

$$\begin{array}{c}
 \frac{}{\Gamma, A \vdash A} \text{ (Hyp)} \\
 \\
 \frac{\Gamma \vdash A \quad \Gamma \vdash B}{\Gamma \vdash A \wedge B} (\wedge I) \qquad \frac{\Gamma \vdash A \wedge B}{\Gamma \vdash A} (\wedge E_1) \\
 \qquad \qquad \qquad \frac{\Gamma \vdash A \wedge B}{\Gamma \vdash B} (\wedge E_2) \\
 \\
 \frac{\Gamma \vdash A}{\Gamma \vdash A \vee B} (\vee I_1) \qquad \frac{\Gamma \vdash A \vee B \quad \Gamma, A \vdash C \quad \Gamma, B \vdash C}{\Gamma \vdash C} (\vee E) \\
 \frac{\Gamma \vdash B}{\Gamma \vdash A \vee B} (\vee I_2) \\
 \\
 \frac{\Gamma, A \vdash B}{\Gamma \vdash A \rightarrow B} (\rightarrow I) \qquad \frac{\Gamma \vdash A \rightarrow B \quad \Gamma \vdash A}{\Gamma \vdash B} (\rightarrow E) \\
 \\
 \frac{\Gamma, A \vdash B \wedge \neg B}{\Gamma \vdash \neg A} (\neg I) \qquad \frac{\Gamma \vdash A \quad \Gamma \vdash \neg A}{\Gamma \vdash B} (\neg E) \\
 \\
 \frac{\Gamma \vdash A}{\Gamma \vdash \neg \neg A} (\neg \neg I) \qquad \frac{\Gamma \vdash \neg \neg A}{\Gamma \vdash A} (\neg \neg E) \\
 \\
 \frac{\Gamma \vdash A[y/x]}{\Gamma \vdash \forall x A} (\forall I) \qquad \frac{\Gamma \vdash \forall x A}{\Gamma \vdash A[t/x]} (\forall E) \\
 \\
 \frac{\Gamma \vdash A[t/x]}{\Gamma \vdash \exists x A} (\exists I) \qquad \frac{\Gamma \vdash \exists x A \quad \Gamma, A[y/x] \vdash B}{\Gamma \vdash B} (\exists E)
 \end{array}$$

In the quantifier rules above, we assume that all substitutions are admissible and y does not occur free in Γ or A .

Rules for Equality (an extension for languages with $=$):

Let t, t_1, t_2 be arbitrary terms and again assume all substitutions are admissible.

$$\frac{}{\Gamma \vdash t = t} (= I) \qquad \frac{\Gamma \vdash t_1 = t_2 \quad \Gamma \vdash A[t_1/x]}{\Gamma \vdash A[t_2/x]} (= E)$$

Note: The $=$ sign is part of the object language, not a meta symbol.