Homework Assignment #4

Note

This assignment is due 1:20PM Thursday, November 27, 2014. Please write or type your answers on A4 (or similar size) paper. Put your completed homework on the instructor's desk before the class starts. For late submissions, please drop them in Yih-Kuen Tsay's mail box on the first floor of Management Building II. A late submission will be penalized by 20% for each working day overdue. You may discuss the problems with others, but copying answers is strictly forbidden.

Problems

1. A majority of an array of n elements is an element that has more than $\frac{n}{2}$ occurrences in the array. Below is a program that finds the majority of an array X of n elements or determines its non-existence. (Hint: if $A[i] \neq A[j]$, then the majority of A remains a majority in a new array B obtained from A by removing $A[i]$ and $A[j]$. Check out Udi Manber's algorithms book if you cannot understand the program.)

```
C,M := X[1],1;
i := 2;
while i<=n do
   if M=0 then C,M := X[i],1
          else if C=X[i] then M := M+1
                         else M := M-1
               fi
   fi;
   i := i+1
od;
if M=0 then Majority := -1
       else Count := 0;
            i := 1;
            while i<=n do
               if X[i]=C then Count := Count+1 fi;
               i := i+1
            od;
            if Count>n/2 then Majority := C
                         else Majority := -1
            fi
```

```
fi
```

(a) Annotate the program into a *standard* proof outline, showing clearly the partial correctness of the program. (40 points)

(b) Prove the validity of the annotation for the first while loop. (20 points)

2. The following fundamental properties are usually taken as axioms for the predicate transformer wp (weakest precondition):

   - **Law of the Excluded Miracle**: $wp(S, \mathit{false}) \equiv \mathit{false}$.
   - **Distributivity of Conjunction**: $wp(S, Q_1) \wedge wp(S, Q_2) \equiv wp(S, Q_1 \wedge Q_2)$.
   - **Distributivity of Disjunction** for deterministic S: $wp(S, Q_1) \vee wp(S, Q_2) \equiv wp(S, Q_1 \vee Q_2)$.

   From the axioms (plus the usual logical and algebraic laws), derive the following properties of wp (Hint: not every axiom is useful):

   - **Law of Monotonicity**: if $Q_1 \Rightarrow Q_2$, then $wp(S, Q_1) \Rightarrow wp(S, Q_2)$. (20 points)
   - **Distributivity of Disjunction** (for any command): $wp(S, Q_1) \vee wp(S, Q_2) \Rightarrow wp(S, Q_1 \vee Q_2)$. (20 points)