Natural Deduction in the Sequent Form
(for Classic Logic)

$$\frac{}{\Gamma, A \vdash A} \, (Hyp)$$

$$\frac{\Gamma \vdash A \quad \Gamma \vdash B}{\Gamma \vdash A \wedge B} \, (\wedge I)$$

$$\frac{\Gamma \vdash A \wedge B}{\Gamma \vdash A} \, (\wedge E_1)$$

$$\frac{\Gamma \vdash A \wedge B}{\Gamma \vdash B} \, (\wedge E_2)$$

$$\frac{\Gamma \vdash A}{\Gamma \vdash A \vee B} \, (\vee I_1)$$

$$\frac{\Gamma \vdash B}{\Gamma \vdash A \vee B} \, (\vee I_2)$$

$$\frac{\Gamma \vdash A \vee B \quad \Gamma, A \vdash C \quad \Gamma, B \vdash C}{\Gamma \vdash C} \, (\vee E)$$

$$\frac{\Gamma, A \vdash B}{\Gamma \vdash A \to B} \, (\to I)$$

$$\frac{\Gamma \vdash A \to B \quad \Gamma \vdash A}{\Gamma \vdash B} \, (\to E)$$

$$\frac{\Gamma, A \vdash B \wedge \neg B}{\Gamma \vdash \neg A} \, (\neg I)$$

$$\frac{\Gamma \vdash A \quad \Gamma \vdash \neg A}{\Gamma \vdash B} \, (\neg E)$$

$$\frac{\Gamma \vdash A}{\Gamma \vdash \neg\neg A} \, (\neg\neg I)$$

$$\frac{\Gamma \vdash \neg\neg A}{\Gamma \vdash A} \, (\neg\neg E)$$

$$\frac{\Gamma \vdash A[y/x]}{\Gamma \vdash \forall x A} \, (\forall I)$$

$$\frac{\Gamma \vdash \forall x A}{\Gamma \vdash A[t/x]} \, (\forall E)$$

$$\frac{\Gamma \vdash A[t/x]}{\Gamma \vdash \exists x A} \, (\exists I)$$

$$\frac{\Gamma \vdash \exists x A \quad \Gamma, A[y/x] \vdash B}{\Gamma \vdash B} \, (\exists E)$$

In the quantifier rules above, we assume that all substitutions are admissible and y does not occur free in Γ, A, or B.

Rules for Equality (an extension for languages with =):

Let t, t_1, t_2 be arbitrary terms and again assume all substitutions are admissible.

$$\frac{}{\Gamma \vdash t = t} \, (= I)$$

$$\frac{\Gamma \vdash t_1 = t_2 \quad \Gamma \vdash A[t_1/x]}{\Gamma \vdash A[t_2/x]} \, (= E)$$

Note: The = sign is part of the object language, not a meta symbol.