Inference Rules of Hoare Logic

$$\overline{\{Q[E/x]\}\ x := E\ \{Q\}}$$ (Assignment)

Note: to treat multiple assignments, view x as a list of distinct variables and E as a list of expressions.

$$\overline{\{Q[(b;i:E)/b]\}\ b[i] := E\ \{Q\}}$$ (Assignment: array)

$$\overline{\{P\}\ \textbf{skip}\ \{P\}}$$ (Skip)

$$\frac{\{P\}\ S_1\ \{Q\}\qquad \{Q\}\ S_2\ \{R\}}{\{P\}\ S_1;S_2\ \{R\}}$$ (Sequence)

$$\frac{\{P \wedge B\}\ S_1\ \{Q\}\qquad \{P \wedge \neg B\}\ S_2\ \{Q\}}{\{P\}\ \textbf{if}\ B\ \textbf{then}\ S_1\ \textbf{else}\ S_2\ \textbf{fi}\ \{Q\}}$$ (Conditional)

"**if** B **then** S **fi**" can be treated as "**if** B **then** S **else skip fi**" or directly with the following rule:

$$\frac{\{P \wedge B\}\ S\ \{Q\}\qquad P \wedge \neg B \to Q}{\{P\}\ \textbf{if}\ B\ \textbf{then}\ S\ \textbf{fi}\ \{Q\}}$$ (If-Then)

$$\frac{\{P \wedge B\}\ S\ \{P\}}{\{P\}\ \textbf{while}\ B\ \textbf{do}\ S\ \textbf{od}\ \{P \wedge \neg B\}}$$ (**while**)

$$\frac{P \to P'\qquad \{P'\}\ S\ \{Q'\}\qquad Q' \to Q}{\{P\}\ S\ \{Q\}}$$ (Consequence)

$$\frac{\text{"}\textbf{proc}\ \text{p}(\textbf{in}\ x;\ \textbf{in out}\ y;\ \textbf{out}\ z);\ \{P\}\ S\ \{Q\};\text{"\ is proved}}{\{P[a,b/x,y] \wedge I)\}\ \text{p}(a,b,c)\ \{Q[b,c/y,z] \wedge I\}}$$ (Procedure Call)

where b, c are (lists of) distinct variables and I does not refer to variables changed by procedure p.

$$\frac{\{P \wedge B\}\ S\ \{P\}\qquad \{P \wedge B \wedge t = Z\}\ S\ \{t < Z\}\qquad P \wedge B \to (t \geq 0)}{\{P\}\ \textbf{while}\ B\ \textbf{do}\ S\ \textbf{od}\ \{P \wedge \neg B\}}$$ (**while**: simply total)

$$\frac{\{P \wedge B\}\ S\ \{P\}\qquad \{P \wedge B \wedge \delta = D\}\ S\ \{\delta \prec D\}\qquad P \wedge B \to (\delta \in W)}{\{P\}\ \textbf{while}\ B\ \textbf{do}\ S\ \textbf{od}\ \{P \wedge \neg B\}}$$ (**while**: well-founded)

Auxiliary Rules:

$$\frac{P \rightarrow P' \qquad \{P'\}\ S\ \{Q\}}{\{P\}\ S\ \{Q\}}$$ (Strengthening Precondition)

$$\frac{\{P\}\ S\ \{Q'\} \qquad Q' \rightarrow Q}{\{P\}\ S\ \{Q\}}$$ (Weakening Postcondition)

$$\frac{\{P_1\}\ S\ \{Q_1\} \qquad \{P_2\}\ S\ \{Q_2\}}{\{P_1 \wedge P_2\}\ S\ \{Q_1 \wedge Q_2\}}$$ (Conjunction)

$$\frac{\{P_1\}\ S\ \{Q_1\} \qquad \{P_2\}\ S\ \{Q_2\}}{\{P_1 \vee P_2\}\ S\ \{Q_1 \vee Q_2\}}$$ (Disjunction)