

## Homework Assignment #0

### Due Time/Date

2:20PM Wednesday, September 13, 2023. The purpose of this homework is for you to get warmed up and will not be counted as part of your grade of this course.

### How to Submit

Please write or type your answers on A4 (or similar size) paper. Put your completed homework on the instructor's desk before the class starts. For late submissions, please drop them in Yih-Kuen Tsay's mail box on the first floor of Management Building 2. You may discuss the problem with others, but copying answers is strictly forbidden.

### Problems

1. Consider the “untangling line segments” problem discussed in class. Your task is to refine the rank function given in class so that it maps program states to *non-negative integers*. Try to be precise (using mathematical concepts and notations). You must show in sufficient details that the execution of an untangling operation, if enabled, will decrease the value of the rank function by *at least one*.
2. Below is a function implementing a variant of Euclid's algorithm:

```
Function originalEuclid( $m, n$ );  
begin  
  // assume that  $m > 0$  and  $n > 0$   
   $x := m$ ;  
   $y := n$ ;  
  while  $x \neq y$  do  
    if  $x < y$  then swap( $x, y$ );  
     $x := x - y$ ;  
  od  
  ...  
end
```

where swap( $x, y$ ) exchanges the values of  $x$  and  $y$ .

- (a) Please give a suitable invariant for the while loop. The loop invariant should be strong enough to determine that, when the program exits the while loop, the value

of  $x$  (or  $y$ ) equals the greatest common divisor of  $m$  and  $n$ . Try to be as precise as possible.

- (b) Please also give a suitable rank function (mapping program states to non-negative integers) for the while loop. Each iteration of the while loop (when the Boolean condition holds) should reduce the value of the rank function by at least one. Again, try to be as precise as possible.