Final

Note

This is an open-book exam. You may consult any books, papers, or notes, but discussion with other students or seeking outside help is strictly forbidden.

Problems

1. Prove, using *Natural Deduction* (in the sequent form) the validity of the following sequents:
   (a) (10 %) $\neg A \wedge \neg B \vdash \neg(A \vee B)$
   (b) (10 %) $A \wedge \exists x B \vdash \exists x(A \wedge B)$, if x does not occur free in A
2. (10 %) The first-order theory for *monoids* contains the following two axioms:
   - $\forall a \forall b \forall c(a \cdot (b \cdot c) = (a \cdot b) \cdot c)$. (Associativity)
   - $\forall a((a \cdot e = a) \wedge (e \cdot a = a))$. (Identity)

   Here e is a constant, called the identity, and $\cdot$ is the binary operation. Let M denote the set of the two axioms. Prove using *Natural Deduction* the validity of the sequent $M \vdash \forall e'(\forall a((a \cdot e' = a) \wedge (e' \cdot a = a)) \rightarrow e' = e)$, which says that the identity element of a monoid is unique. (Hint: a typical proof in algebra books is the following: assuming e' is an identity, $e' = e' \cdot e = e$.)
3. The program segment below solves the following problem: given a sequence $x_1, x_2, \cdots, x_n$ of real numbers (not necessarily positive), represented as an array X, find a subsequence $x_i,\ x_{i+1},\ \cdots,\ x_j$ (of consecutive elements) such that the sum of the numbers in it is maximum over all subsequences of consecutive elements. (Note: the program in fact gives only the sum, rather than the indices of the first and the last elements, of the maximum subsequence.)

$G\_Max := 0$;
$S\_Max := 0$;
$i := 1$;
while $i \leq n$ **do**
  if $S\_Max + x[i] > G\_Max$ **then**
    $S\_Max := S\_Max + x[i]$;
    $G\_Max := S\_Max$
  else if $x[i] + S\_Max > 0$ **then**

$S\_Max := S\_Max + x[i]$

else $S\_Max := 0$

fi

fi

$i := i + 1$

od

(a) (10 %) Give a pair of pre and post-conditions that precisely describe the requirements for the program segment.

(b) (20 %) Annotate the program segment into a proof outline that clearly shows the correctness of the program.

4. (20 %) Prove the partial correctness of the following program using the Owicki-Gries method.

$$\{true\}$$
$$acc := 0;$$
$$Q_0, Q_1 := false, false;$$
$$\left[\begin{array}{lcl} Q_0 := true; & & Q_1 := true; \\ T := 0; & & T := 1; \\ \textbf{await } \neg Q_1 \vee (T \neq 0); & \| & \textbf{await } \neg Q_0 \vee (T \neq 1); \\ s_0 := acc; & & s_1 := acc; \\ acc := s_0 + 1; & & acc := s_1 + 1; \\ Q_0 := false; & & Q_1 := false; \end{array}\right]$$
$$\{acc = 2\}$$

5. (20 %) Assuming that the leads-to operator in UNITY is defined without the disjunction rule, prove the following derived rule.

$$\frac{p \mapsto q \qquad p' \mapsto q'}{p \vee p' \mapsto q \vee q'}$$