Final

Note

This is an open-book exam. You may consult any books, papers, or notes, but discussion with other students or seeking outside help is strictly forbidden.

Problems

1. Prove, using *Natural Deduction*, the validity of the following sequents:

   (a) (10 %) $\vdash A \wedge (B \wedge C) \rightarrow (A \wedge B) \wedge C$

   (b) (10 %) $\vdash \neg(A \wedge \neg A)$

2. The program segment below solves the following problem: given a sequence x_1, x_2, $\cdots$, x_n of real numbers (not necessarily positive), represented as an array X, find a subsequence x_i, x_{i+1}, $\cdots$, x_j (of consecutive elements) such that the sum of the numbers in it is maximum over all subsequences of consecutive elements. (Note: the program in fact gives only the indices of the first and the last elements of the subsequence.)

```
G_Max := 0;
S_Max := 0;
i := 1;
while i ≤ n do
    if S_Max + x[i] > G_Max then
       S_Max := S_Max + x[i];
       G_Max := S_Max
    else if x[i] + S_Max > 0 then
             S_Max := S_Max + x[i]
         else S_Max := 0
         fi
    fi
    i := i + 1
od
```

   (a) (10 %) Give a pair of pre and post-conditions that precisely describe the requirements for the program segment.

   (b) (20 %) Annotate the program segment into a proof outline, showing clearly the correctness of the program.

3. Solve the following problems for fair transition systems, which we have studied as a model for concurrent reactive systems. You may consider only justice, and ignore compassion, constraints.

   (a) (20 %) Give a suitable formal definition for *open* fair transition systems, or fair transition modules, where the set of variables is partitioned into *in* and *out* variables. A system reads from, but does not write on, its *in* variables. The environment of an open system reads from, but does not write on, the *out* variables of the system. The computation of an open system should take into account the interference from its environment.

   (b) (10 %) Define a parallel composition operation "$\|$" on two open fair transition systems that follows the interleaving model of concurrency. The parallel composition of two open systems is another open system. Be careful about the condition under which two systems may be composed.

   (c) (20 %) For two systems S_1 and S_2 that are composable, prove that the set of computations of $S_1 \,\|\, S_2$, namely $Comp(S_1 \,\|\, S_2)$, is the intersection of $Comp(S_1)$ and $Comp(S_2)$. (Note: adjust your definitions in the preceding sub-problems so that this compositional property holds.)