## Homework Assignment \#9

## Note

This assignment is due 2:10PM Wednesday, May 22, 2013. Please write or type your answers on A4 (or similar size) paper. Drop your homework by the due time in Yih-Kuen Tsay's mail box on the first floor of Management College Building II, or put it on the instructor's desk before the class on the due date starts. Late submission will be penalized by $20 \%$ for each working day overdue. You may discuss the problems with others, but copying answers is strictly forbidden.

## Problems

(Note: problems marked with "Exercise X.XX" or "Problem X.XX" are taken from [Sipser 2006] with probable adaptation.)

1. (Exercise $4.6 ; 10$ points) Let $B$ be the set of all infinite sequences over $\{0,1\}$. Show that $B$ is uncountable, using a proof by diagonalization.
2. (Exercise 4.8; 10 points) Review the way that we define sets to be of the same size in Definition 4.12. Show that "are of the same size" is an equivalence relation.
3. (Problem 4.16; 20 points) Prove that $E Q_{\text {DFA }}$ is decidable by testing the two DFAs on all strings up to a certain size. Calculate a size that works.
4. (Problem 4.28; 20 points) Let $A$ be a Turing-recognizable language consisting of descriptions of Turing machines, $\left\{\left\langle M_{1}\right\rangle,\left\langle M_{2}\right\rangle, \ldots\right\}$, where every $M_{i}$ is a decider. Prove that some decidable language $D$ is not decided by any decider $M_{i}$ whose description appears in $A$. (Hint: You may find it helpful to consider an enumerator for $A$.)
5. (Exercise 5.1; 20 points) Show that $E Q_{\mathrm{CFG}}$ is undecidable.
6. (Exercise $5.4 ; 20$ points) If $A$ is reducible to $B$ and $B$ is a regular language, does that imply that $A$ is a regular language? Why or why not?
