## Homework Assignment \#9

## Note

This assignment is due $2: 10 \mathrm{PM}$ Wednesday, June 3, 2015. Please write or type your answers on A4 (or similar size) paper. Drop your homework by the due time in Yih-Kuen Tsay's mail box on the first floor of Management College Building II, or put it on the instructor's desk before the class on the due date starts. Late submission will be penalized by $20 \%$ for each working day overdue. You may discuss the problems with others, but copying answers is strictly forbidden.

## Problems

(Note: problems marked with "Exercise X.XX" or "Problem X.XX" are taken from [Sipser 2013] with probable adaptation.)

1. (Problem 4.12; 10 points) Let $A$ be a Turing-recognizable language consisting of descriptions of Turing machines, $\left\{\left\langle M_{1}\right\rangle,\left\langle M_{2}\right\rangle, \ldots\right\}$, where every $M_{i}$ is a decider. Prove that some decidable language $D$ is not decided by any decider $M_{i}$ whose description appears in $A$. (Hint: You may find it helpful to consider an enumerator for $A$.)
2. (Problem 4.22; 20 points) Let $A$ and $B$ be two disjoint languages. Say that language $C$ separates $A$ and $B$ if $A \subseteq C$ and $B \subseteq \bar{C}$. Show that any two disjoint co-Turingrecognizable languages are separable by some decidable language.
3. (Problem 4.25; 10 points) Prove that $E Q_{\text {DFA }}$ is decidable by testing the two DFAs on all strings up to a certain size. Calculate a size that works.
4. (Problem 4.31; 20 points) Let $I N F I N I T E_{\mathrm{PDA}}=\{\langle M\rangle \mid M$ is a PDA and $L(M)$ is infinite $\}$. Show that INFINITE $_{\mathrm{PDA}}$ is decidable.
5. (Exercise 5.1; 20 points) Show that $E Q_{\mathrm{CFG}}$ is undecidable.
6. (Exercise $5.4 ; 20$ points) If $A$ is reducible to $B$ and $B$ is a regular language, does that imply that $A$ is a regular language? Why or why not?
