Homework Assignment #8

Note

This assignment is due 2:10PM Wednesday, May 25, 2016. Please write or type your answers on A4 (or similar size) paper. Drop your homework by the due time in Yih-Kuen Tsay's mail box on the first floor of Management College Building II, or put it on the instructor's desk before the class on the due date starts. Late submission will be penalized by 20% for each working day overdue. You may discuss the problems with others, but copying answers is strictly forbidden.

Problems

(Note: problems marked with "Exercise X.XX" or "Problem X.XX" are taken from [Sipser 2013] with probable adaptation.)

1. (10 points) Give a formal definition of a Turing machine that appends a # at the end of the input string and then copies and appends the original input after the #. The input alphabet is $\{0, 1\}$.

2. (Exercise 3.4; 10 points) Give a formal definition of an enumerator. Consider it to be a type of two-tape Turing machine that uses its second tape as the printer. Include a definition of the enumerated language.

3. (Problem 3.10; 20 points) Let $c_1x^n + c_2x^{n-1} + \cdots + c_nx + c_{n+1}$ be a polynomial with a root at $x = x_0$. Let $c_{\max}$ be the largest absolute value of a c_i. Show that

$$|x_0| < (n+1)\frac{c_{\max}}{|c_1|}.$$

4. (Problem 3.11; 20 points) Show that single-tape TMs that cannot write on the portion of the tape containing the input string recognize only regular languages.

5. (Problem 3.13; 20 points) Show that a language is decidable iff some enumerator enumerates the language in the standard string order.

6. (10 points) Let $A = \{\langle M, N\rangle \mid M \text{ is a PDA, } N \text{ is a DFA, and } L(M) \subseteq L(N)\}$. Show that A is decidable.

7. (Problem 4.4; 10 points) Let $A\varepsilon_{\text{CFG}} = \{\langle G\rangle \mid G \text{ is a CFG that generates } \varepsilon\}$. Show that $A\varepsilon_{\text{CFG}}$ is decidable.