Homework Assignment #10

Note

This assignment is due 2:10PM Wednesday, June 7, 2017. Please write or type your answers on A4 (or similar size) paper. Drop your homework by the due time in Yih-Kuen Tsay's mail box on the first floor of Management College Building 2, or put it on the instructor's desk before the class on the due date starts. Late submission will be penalized by 20% for each working day overdue. You may discuss the problems with others, but copying answers is strictly forbidden.

Problems

(Note: problems marked with "Exercise X.XX" or "Problem X.XX" are taken from [Sipser 2013] with probable adaptation.)

1. (Problem 5.9; 10 points) Let $AMBIG_{\text{CFG}} = \{\langle G \rangle \mid G \text{ is an ambiguous CFG}\}$. Show that $AMBIG_{\text{CFG}}$ is undecidable. (Hint: use a reduction from PCP. Given an instance
$$P = \left\{ \left[\frac{t_1}{b_1} \right], \left[\frac{t_2}{b_2} \right], \dots, \left[\frac{t_k}{b_k} \right] \right\}$$
of PCP, construct a CFG G with the rules:
$$\begin{array}{lcl} S & \rightarrow & T \mid B \\ T & \rightarrow & t_1 T a_1 \mid \cdots \mid t_k T a_k \mid t_1 a_1 \mid \cdots \mid t_k a_k \\ B & \rightarrow & t_1 B a_1 \mid \cdots \mid t_k B a_k \mid t_1 a_1 \mid \cdots \mid t_k a_k, \end{array}$$
where $a_1, \dots, a_k$ are new terminal symbols. Prove that this reduction works.)

2. (Problem 5.14(b); 20 points) Define a *two-headed finite automaton* (2DFA) to be a deterministic finite automaton that has two read-only, bidirectional heads that start at the left-hand end of the input tape and can be independently controlled to move in either direction. The tape of a 2DFA is finite and is just large enough to contain the input plus two additional blank tape cells, one on the left-end and one on the right-hand end, that serve as delimiters. A 2DFA accepts its input by entering a special accept state. For example, a 2DFA can recognize the language $\{a^n b^n c^n \mid n \geq 0\}$.

   Let $E_{\text{2DFA}} = \{\langle M \rangle \mid M \text{ is a 2DFA and } L(M) = \emptyset\}$. Show that E_{2DFA} is undecidable.

3. (Problem 5.18(b); 10 points) Use Rice's theorem to prove the undecidability of the language $\{\langle M \rangle \mid M \text{ is a TM and } 1011 \in L(M)\}$.

4. (Problem 5.29; 20 points) A *useless state* in a Turing machine is one that is never entered on any input string. Consider the problem of determining whether a Turing machine has any useless states. Formulate this problem as a language and show that it is undecidable.

5. (Problem 5.34; 20 points) Show that the Post Correspondence Problem is undecidable over the binary alphabet $\Sigma = \{0, 1\}$.

6. (Problem 5.36; 20 points) Prove that there exists an undecidable subset of $\{1\}^*$.