Homework Assignment #9

Note

This assignment is due 2:10PM Wednesday, May 31, 2017. Please write or type your answers on A4 (or similar size) paper. Drop your homework by the due time in Yih-Kuen Tsay's mail box on the first floor of Management College Building 2, or put it on the instructor's desk before the class on the due date starts. Late submission will be penalized by 20% for each working day overdue. You may discuss the problems with others, but copying answers is strictly forbidden.

Problems

(Note: problems marked with "Exercise X.XX" or "Problem X.XX" are taken from [Sipser 2013] with probable adaptation.)

1. (Exercise 4.7; 10 points) Let B be the set of all infinite sequences over $\{0, 1\}$. Show that B is uncountable, using a proof by diagonalization.

2. (Problem 4.12; 10 points) Let A be a Turing-recognizable language consisting of descriptions of Turing machines, $\{\langle M_1 \rangle, \langle M_2 \rangle, \ldots\}$, where every M_i is a decider. Prove that some decidable language D is not decided by any decider M_i whose description appears in A. (Hint: You may find it helpful to consider an enumerator for A.)

3. (Problem 4.14; 20 points) Let $C = \{\langle G, x \rangle \mid G$ is a CFG and x is a substring of some $y \in L(G)\}$. Show that C is decidable. (Hint: an elegant solution to this problem uses the decider for E_{CFG}.)

4. (Problem 4.18; 20 points) A *useless state* in a pushdown automaton is never entered on any input string. Consider the problem of determining whether a pushdown automaton has any useless states. Formulate this problem as a language and show that it is decidable.

5. (Exercise 5.1; 20 points) Show that EQ_{CFG} is undecidable.

6. (Exercise 5.4; 20 points) If A is reducible to B and B is a regular language, does that imply that A is a regular language? Why or why not?