Homework Assignment #5

Due Time/Date

This assignment is due 2:10PM Tuesday, April 14, 2020. Late submission will be penalized by 20% for each working day overdue.

How to Submit

Please use a word processor or scan hand-written answers to produce a single PDF file. Name your file according to this pattern: "b057050xx-hw5". Upload the PDF file to the Ceiba course site for Theory of Computing 2020: https://ceiba.ntu.edu.tw/1082theory2020. You may discuss the problems with others, but copying answers is strictly forbidden.

Problems

(Note: problems marked with "Exercise X.XX" or "Problem X.XX" are taken from [Sipser 2006, 2013] with probable adaptation.)

1. (Exercise 2.1; 20 points) Consider the following CFG discussed in class, where for convenience the variables have been renamed with single letters.

$$\begin{array}{lcl} E & \to & E+T \mid T \\ T & \to & T \times F \mid F \\ F & \to & (E) \mid a \end{array}$$

   Give (leftmost) derivations and the corresponding parse trees for the following strings.

   (a) $(a+a) \times a$

   (b) $(a+(a))$

2. (Exercise 2.4; 20 points) Give context-free grammars that generate the following languages. In all parts the alphabet Σ is $\{0,1\}$.

   (a) $\{w \mid \text{the length of } w \text{ is odd}\}$

   (b) $\{w \mid w = w^R, \text{ that is, } w \text{ is a palindrome}\}$

3. (Exercise 2.6d; 10 points) Give a context-free grammar that generates the language $\{x_1\#x_2\#\cdots\#x_k \mid k \geq 1, \text{ each } x_i \in \{a,b\}^*, \text{ and for some } i \text{ and } j,\ x_i = x_j^R\}$.

4. (Exercise 2.8; 10 points) Show that the string "`the girl touches a boy with the flower`" has two different leftmost derivations in the following CFG.

$$
\begin{array}{rcl}
\langle \text{SENTENCE} \rangle & \to & \langle \text{NOUN-PHRASE} \rangle \langle \text{VERB-PHRASE} \rangle \\
\langle \text{NOUN-PHRASE} \rangle & \to & \langle \text{CMPLX-NOUN} \rangle \mid \\
 & & \langle \text{CMPLX-NOUN} \rangle \langle \text{PREP-PHRASE} \rangle \\
\langle \text{VERB-PHRASE} \rangle & \to & \langle \text{CMPLX-VERB} \rangle \mid \\
 & & \langle \text{CMPLX-VERB} \rangle \langle \text{PREP-PHRASE} \rangle \\
\langle \text{PREP-PHRASE} \rangle & \to & \langle \text{PREP} \rangle \langle \text{CMPLX-NOUN} \rangle \\
\langle \text{CMPLX-NOUN} \rangle & \to & \langle \text{ARTICLE} \rangle \langle \text{NOUN} \rangle \\
\langle \text{CMPLX-VERB} \rangle & \to & \langle \text{VERB} \rangle \mid \langle \text{VERB} \rangle \langle \text{NOUN-PHRASE} \rangle \\
\langle \text{ARTICLE} \rangle & \to & \texttt{a} \mid \texttt{the} \\
\langle \text{NOUN} \rangle & \to & \texttt{boy} \mid \texttt{girl} \mid \texttt{flower} \\
\langle \text{VERB} \rangle & \to & \texttt{touches} \mid \texttt{likes} \mid \texttt{sees} \\
\langle \text{PREP} \rangle & \to & \texttt{with}
\end{array}
$$

5. (Exercise 2.9; 20 points) Give a context-free grammar that generates the language

$$A = \{a^i b^j c^k \mid i = j \text{ or } j = k \text{ where } i, j, k \geq 0\}.$$

   Is your grammar ambiguous? Why or why not?

6. (Exercise 2.14; 20 points) Convert the following CFG (where A is the start variable) into an equivalent CFG in Chomsky normal form, using the procedure given in Theorem 2.9.

$$
\begin{array}{rcl}
A & \to & BAB \mid B \mid \varepsilon \\
B & \to & 01 \mid \varepsilon
\end{array}
$$