# Homework Assignment #9

## Due Time/Date

This assignment is due 2:10PM Tuesday, June 1, 2021. Late submission will be penalized by 20% for each working day overdue.

## How to Submit

Please use a word processor or scan hand-written answers to produce a single PDF file. Name your file according to this pattern: "b067050xx-hw9". Upload the PDF file to the NTU COOL course site for Theory of Computing 2021. You may discuss the problems with others, but copying answers is strictly forbidden.

## Problems

(Note: problems marked with "Exercise X.XX" or "Problem X.XX" are taken from [Sipser 2013] with probable adaptation.)

1. (Problem 4.12; 10 points) Let $A$ be a Turing-recognizable language consisting of descriptions of Turing machines, $\{\langle M_1\rangle, \langle M_2\rangle, \ldots\}$, where every $M_i$ is a decider. Prove that some decidable language $D$ is not decided by any decider $M_i$ whose description appears in $A$. (Hint: you may find it helpful to consider an enumerator for $A$.)

2. (Problem 4.14; 20 points) Let $C = \{\langle G, x\rangle \mid G \text{ is a CFG and } x \text{ is a substring of some } y \in L(G)\}$. Show that $C$ is decidable. (Hint: an elegant solution to this problem uses the decider for $E_{\text{CFG}}$.)

3. (Problem 4.22; 20 points) Let $A$ and $B$ be two disjoint languages. Say that language $C$ *separates* $A$ and $B$ if $A \subseteq C$ and $B \subseteq \overline{C}$. Show that any two disjoint co-Turing-recognizable languages are separable by some decidable language.

4. (Problem 4.31; 20 points) Let $\mathit{INFINITE}_{\text{PDA}} = \{\langle M\rangle \mid M \text{ is a PDA and } L(M) \text{ is infinite}\}$. Show that $\mathit{INFINITE}_{\text{PDA}}$ is decidable.

5. (Exercise 5.1; 10 points) Show that $EQ_{\text{CFG}}$ is undecidable.

6. (Exercise 5.4; 20 points) If $A$ is reducible to $B$ and $B$ is a regular language, does that imply that $A$ is a regular language? Why or why not?