Homework Assignment #5

Due Time/Date

This assignment is due 2:20PM Tuesday, April 12, 2022. Late submission will be penalized by 20% for each working day overdue.

Note

Please write or type your answers on A4 (or similar size) paper. Drop your homework by the due time in Yih-Kuen Tsay's mail box on the first floor of Management College Building 2, or put it on the instructor's desk before the class on the due date starts. You may discuss the problems with others, but copying answers is strictly forbidden.

Problems

(Note: problems marked with "Exercise X.XX" or "Problem X.XX" are taken from [Sipser 2006, 2013] with probable adaptation.)

1. (Exercise 2.1; 20 points) Consider the following CFG discussed in class, where for convenience the variables have been renamed with single letters.

$$\begin{array}{lcl} E & \rightarrow & E + T \mid T \\ T & \rightarrow & T \times F \mid F \\ F & \rightarrow & (E) \mid a \end{array}$$

   Give (leftmost) derivations and the corresponding parse trees for the following strings.

   (a) $a \times (a + a)$

   (b) $((a) + a)$

2. (Exercise 2.4; 20 points) Give context-free grammars that generate the following languages. In all parts the alphabet Σ is $\{0, 1\}$.

   (a) $\{w \mid \text{the length of } w \text{ is odd}\}$

   (b) $\{w \mid w = w^R, \text{ that is, } w \text{ is a palindrome}\}$

3. (Exercise 2.6d; 10 points) Give a context-free grammar that generates the language $\{x_1\#x_2\#\cdots\#x_k \mid k \geq 1, \text{ each } x_i \in \{a, b\}^*, \text{ and for some } i \text{ and } j,\ x_i = x_j^R\}$.

4. (Exercise 2.8; 10 points) Show that the string "`the boy likes the girl with a flower`" has two different leftmost derivations in the following CFG.

$$
\begin{array}{rcl}
\langle\text{SENTENCE}\rangle & \to & \langle\text{NOUN-PHRASE}\rangle\langle\text{VERB-PHRASE}\rangle \\
\langle\text{NOUN-PHRASE}\rangle & \to & \langle\text{CMPLX-NOUN}\rangle \mid \\
 & & \langle\text{CMPLX-NOUN}\rangle\langle\text{PREP-PHRASE}\rangle \\
\langle\text{VERB-PHRASE}\rangle & \to & \langle\text{CMPLX-VERB}\rangle \mid \\
 & & \langle\text{CMPLX-VERB}\rangle\langle\text{PREP-PHRASE}\rangle \\
\langle\text{PREP-PHRASE}\rangle & \to & \langle\text{PREP}\rangle\langle\text{CMPLX-NOUN}\rangle \\
\langle\text{CMPLX-NOUN}\rangle & \to & \langle\text{ARTICLE}\rangle\langle\text{NOUN}\rangle \\
\langle\text{CMPLX-VERB}\rangle & \to & \langle\text{VERB}\rangle \mid \langle\text{VERB}\rangle\langle\text{NOUN-PHRASE}\rangle \\
\langle\text{ARTICLE}\rangle & \to & \texttt{a} \mid \texttt{the} \\
\langle\text{NOUN}\rangle & \to & \texttt{boy} \mid \texttt{girl} \mid \texttt{flower} \\
\langle\text{VERB}\rangle & \to & \texttt{touches} \mid \texttt{likes} \mid \texttt{sees} \\
\langle\text{PREP}\rangle & \to & \texttt{with}
\end{array}
$$

5. (Exercise 2.9; 20 points) Give a context-free grammar that generates the language

$$A = \{a^i b^j c^k \mid i = j \text{ or } j = k \text{ where } i, j, k \geq 0\}.$$

Is your grammar ambiguous? Why or why not?

6. (Exercise 2.14; 20 points) Convert the following CFG (where A is the start variable) into an equivalent CFG in Chomsky normal form, using the procedure given in Theorem 2.9.

$$
\begin{array}{rcl}
A & \to & BAB \mid B \mid \varepsilon \\
B & \to & 0B1 \mid \varepsilon
\end{array}
$$