Homework Assignment #5

Due Time/Date

This assignment is due 1:20PM Tuesday, April 11, 2023. Late submission will be penalized by 20% for each working day overdue.

Note

Please write or type your answers on A4 (or similar size) paper. Drop your homework by the due time in Yih-Kuen Tsay's mail box on the first floor of Management College Building 2, or put it on the instructor's desk before the class on the due date starts. You may discuss the problems with others, but copying answers is strictly forbidden.

Problems

(Note: problems marked with "Exercise X.XX" or "Problem X.XX" are taken from [Sipser 2006, 2013] with probable adaptation.)

1. (Exercise 2.1; 10 points) Consider the following CFG discussed in class, where for convenience the variables have been renamed with single letters.

$$
\begin{aligned}
E &\rightarrow E + T \mid T \\
T &\rightarrow T \times F \mid F \\
F &\rightarrow (E) \mid a
\end{aligned}
$$

Give (leftmost) derivations and the corresponding parse trees for the following strings.

(a) $(a + a) \times a$

(b) $((a) + a)$

2. (Exercise 2.4; 10 points) Give CFGs that generate the following languages. In all parts the alphabet Σ is $\{0, 1\}$.

(a) $\{w \mid$ the length of w is odd$\}$

(b) $\{w \mid w = w^R,$ that is, w is a palindrome$\}$

3. (Exercise 2.6b adapted; 10 points) Let $\Sigma = \{a, b\}$. Give a CFG that generates the complement of the language $\{a^n b^n \mid n \geq 0\}$. Please make the CFG as simple as possible and explain the intuition behind it.

4. (Problem 2.33; 20 points) Let $\Sigma = \{a, b\}$. Give a CFG generating the language of strings with twice as many a's as b's (no restriction is imposed on the order in which the input symbols may appear). Prove that the CFG is correct.

5. (Exercise 2.8; 10 points) Show that the string "`the boy likes the girl with a flower`" has two different leftmost derivations in the following CFG.

$$
\begin{aligned}
\langle\text{SENTENCE}\rangle &\rightarrow \langle\text{NOUN-PHRASE}\rangle\langle\text{VERB-PHRASE}\rangle \\
\langle\text{NOUN-PHRASE}\rangle &\rightarrow \langle\text{CMPLX-NOUN}\rangle \mid \\
&\quad\ \langle\text{CMPLX-NOUN}\rangle\langle\text{PREP-PHRASE}\rangle \\
\langle\text{VERB-PHRASE}\rangle &\rightarrow \langle\text{CMPLX-VERB}\rangle \mid \\
&\quad\ \langle\text{CMPLX-VERB}\rangle\langle\text{PREP-PHRASE}\rangle \\
\langle\text{PREP-PHRASE}\rangle &\rightarrow \langle\text{PREP}\rangle\langle\text{CMPLX-NOUN}\rangle \\
\langle\text{CMPLX-NOUN}\rangle &\rightarrow \langle\text{ARTICLE}\rangle\langle\text{NOUN}\rangle \\
\langle\text{CMPLX-VERB}\rangle &\rightarrow \langle\text{VERB}\rangle \mid \langle\text{VERB}\rangle\langle\text{NOUN-PHRASE}\rangle \\
\langle\text{ARTICLE}\rangle &\rightarrow \texttt{a} \mid \texttt{the} \\
\langle\text{NOUN}\rangle &\rightarrow \texttt{boy} \mid \texttt{girl} \mid \texttt{flower} \\
\langle\text{VERB}\rangle &\rightarrow \texttt{touches} \mid \texttt{likes} \mid \texttt{sees} \\
\langle\text{PREP}\rangle &\rightarrow \texttt{with}
\end{aligned}
$$

6. (Exercise 2.9; 20 points) Give a CFG that generates the language

$$A = \{a^i b^j c^k \mid i = j \text{ or } j = k \text{ where } i, j, k \geq 0\}.$$

Is your grammar ambiguous? Why or why not?

7. (Exercise 2.14; 20 points) Convert the following CFG (where A is the start variable) into an equivalent CFG in Chomsky normal form, using the procedure given in Theorem 2.9.

$$
\begin{aligned}
A &\rightarrow BAB \mid B \mid \varepsilon \\
B &\rightarrow 0B1 \mid \varepsilon
\end{aligned}
$$