Homework Assignment #1

Due Time/Date

This assignment is due 2:20PM Tuesday, March 5, 2024. Late submission will be penalized by 20% for each working day overdue.

How to Submit

Please use a word processor or scan hand-written answers to produce a single PDF file and name the file according to this pattern: "b107050xx-hw1". Upload the PDF file to the NTU COOL site for this course. You may discuss the problems with others, but copying answers is strictly forbidden.

Problems

(Note: problems marked with "Exercise X.XX" or "Problem X.XX" are taken from [Sipser 2013] with probable adaptation.)

1. (Exercise 0.7; 30 points) For each part, give a binary relation that satisfies the condition. *Please illustrate the relation using a directed graph.*
   (a) Reflexive and symmetric but not transitive
   (b) Reflexive and transitive but not symmetric
   (c) Symmetric and transitive but not reflexive
2. (20 points) For each part, determine whether the binary relation on the set of integers or reals is an equivalence relation. If it is, please provide a proof; otherwise, please give a counterexample.
   (a) For a fixed non-zero divisor, the two numbers have the same remainder. (Note: for instance, suppose 2 is the divisor. Numbers 4 and 6 have the same remainder, while 4 and 5 do not.)
   (b) The two real numbers are approximately equal. Note: it is up to you to define the notion of "approximately equal" more precisely, but it must not be the same as exactly equal.
3. (20 points) In class, following Sipser's book, we first studied the formal definition of a function and then treated relations as special cases of functions. Please give instead a direct definition of relations and then define functions as special cases of relations. Your definitions should cover the arity of a relation or function and also the meaning of the notation $f(a) = b$.
4. (Problem 0.10; 20 points) Show that every graph having two or more nodes contains two nodes with the same degree. (Note: we assume that every graph is simple and finite, unless explicitly stated otherwise.)

5. (10 points) Consider a round-robin tournament among n players. In the tournament, each player plays once against all other $n-1$ players. There are no draws, i.e., for a match between p and p', the result is either p beat p' or p' beat p. Prove *by induction* that, after a round-robin tournament, it is always possible to arrange the n players in an order $p_1, p_2, \ldots, p_n$ such that p_1 beat p_2, p_2 beat p_3, $\cdots$, and p_{n-1} beat p_n. (Note: the "beat" relation, unlike "$\geq$", is not transitive.)