Homework Assignment #4

Due Time/Date

This assignment is due 2:20PM Tuesday, March 26, 2024. Late submission will be penalized by 20% for each working day overdue.

How to Submit

Please use a word processor or scan hand-written answers to produce a single PDF file and name the file according to this pattern: “b107050xx-hw4”. Upload the PDF file to the NTU COOL site for this course. You may discuss the problems with others, but copying answers is strictly forbidden.

Problems

(Note: problems marked with “Exercise X.XX” or “Problem X.XX” are taken from [Sipser 2013] with probable adaptation.)

1. (Problem 1.43; 10 points) An *all*-NFA M is a 5-tuple $(Q, \Sigma, \delta, q, F)$ that accepts $x \in \Sigma^*$ if every possible state that M could be after reading input x is a state from F. Note, in contrast, that an ordinary NFA accepts a string if *some* state among these possible states is an accept state. Prove that all-NFAs recognize the class of regular languages.

2. (Problem 1.66; 20 points) Let $M = (Q, \Sigma, \delta, q_0, F)$ be a DFA and let h be a state of M called its “home”. A *synchronizing sequence* for M and h is a string $s \in \Sigma^*$ where $\delta(q, s) = h$ for every $q \in Q$. Say that M is *synchronizable* if it has a synchronizing sequence for some state h. Prove that, if M is a k-state synchronizable DFA, then it has a synchronizing sequence of length at most k^3. (Note: $\delta(q, s)$ equals the state where M ends up, when M starts from state q and reads input s.)

3. (Problem 1.61; 20 points) Let the *rotational closure* of language A be $RC(A) = \{yx \mid xy \in A\}$.
   (a) Show that, for any language A, we have $RC(A) = RC(RC(A))$ (i.e., rotational closure, as an operation/function, is idempotent).
   (b) Show that the class of regular languages is closed under rotational closure.

4. (Problem 1.64; 20 points) If A is any language, let $A_{\frac{1}{2}-}$ be the set of all first halves of strings in A so that
$$A_{\frac{1}{2}-} = \{x \mid \text{ for some } y, |x| = |y| \text{ and } xy \in A\}.$$
Show that if A is regular, then so is $A_{\frac{1}{2}-}$.

5. (Problem 1.40; 10 points) Let

$$\Sigma_2 = \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}.$$

Here, Σ_2 contains all columns of `0`s and `1`s of length two. A string of symbols in Σ_2 gives two rows of `0`s and `1`s.

Consider the top and bottom rows to be strings of `0`s and `1`s and let

$$E = \{w \in \Sigma_2^* \mid \text{the bottom row of } w \text{ is the reverse of the top row of } w\}.$$

Show that E is not regular.

6. (Problem 1.71; 20 points) Let $\Sigma = \{0, 1\}$.

   (a) Let $A = \{1^k x \mid x \in \Sigma^* \text{ and } x \text{ contains at least } k \text{ 1s, for } k \geq 1\}$. Show that A is regular.

   (b) Let $B = \{1^k x \mid x \in \Sigma^* \text{ and } x \text{ contains at most } k \text{ 1s, for } k \geq 1\}$. Show that B is not regular.